BUDDHA SERIES

(Unit Wise Solved Question & Answers)

Course – B.Tech (ECE)

College – Buddha Institute of Technology

(AKTU CODE-525)

Department: Electronics and Communication Engineering

Subject: Digital Signal Processing

(BEC 503)

Faculty Name: Anil Singh

UNIT-1

(Realization of Digital Systems)

2 marks Questions:

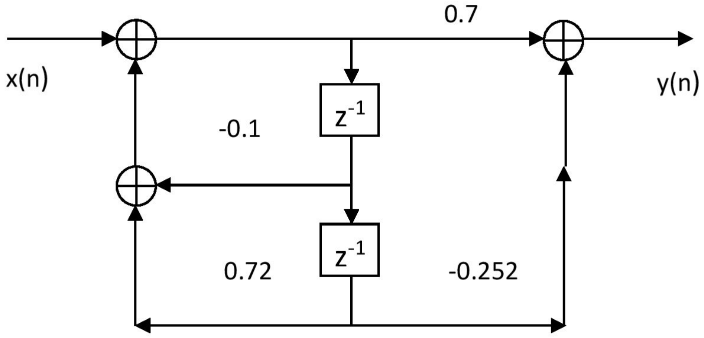
1. Obtain the Direct form-II structure for the system

y(n)= -0.1 y(n-1)+0.72 y(n-2)+0.7 x(n)-0.252 x(n-2).

Soln. For the given system

$$H(z) = \frac{-0.252z^{-2} + 0.7}{0.72z^{-2} + 0.1z^{-1} + 1}$$

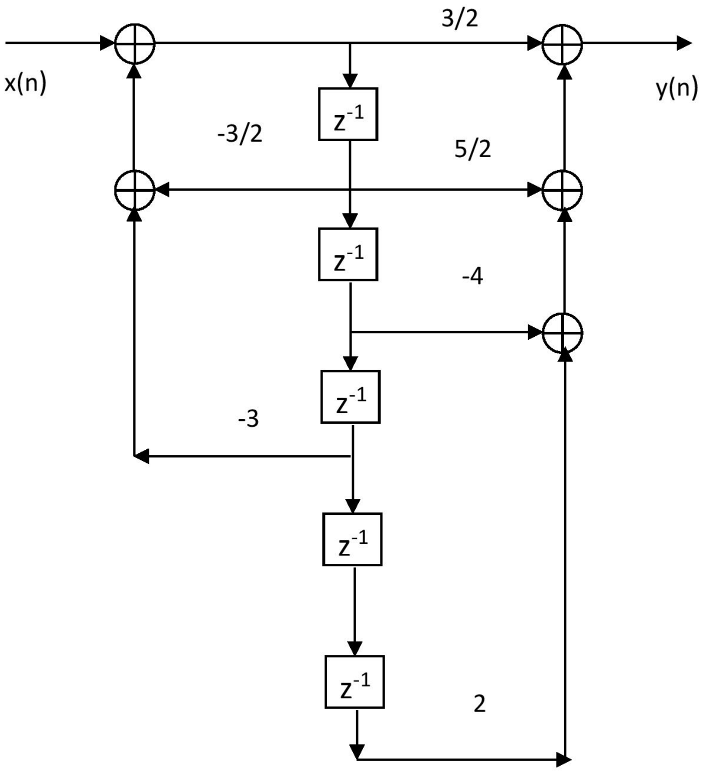
Realization:



2. Develop a canonic direct form realization of the transfer function

$$H(z) = \frac{4z^{-5} - 8z^{-2} + 5z^{-1} + 3}{6z^{-3} + 3z^{-1} + 2}.$$

Sonl.



3. Whether a system represented by its transfer function given by $H(z) = 4 + \frac{3z}{z-(\frac{1}{2})} - \frac{1}{z-(\frac{1}{4})}$, represent an IIR filter.

Soln. Given System

$$H(z) = 4 + \frac{3z}{z-(\frac{1}{2})} - \frac{1}{z-(\frac{1}{4})}$$

The above system can be simplified as

$$H(z) = \frac{z^{-2} - \frac{19}{4}z^{-1} + 7}{\frac{1}{8}z^{-2} - \frac{3}{4}z^{-1} + 1}$$

which is the form of a general IIR system thus the given system represents an IIR filter

4. Consider a causal LTI system whose system function is:

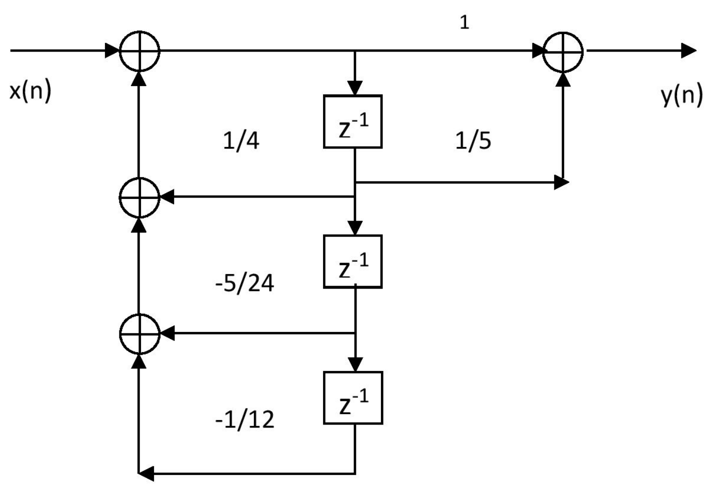
$$H(z) = \frac{1 + (\frac{1}{5})z^{-1}}{(\left(\frac{1}{3}\right)z^{-2} - \left(\frac{1}{2}\right)z^{-1} + 1)\ (1 + \left(\frac{1}{4}\right)z^{-1})}$$

Draw the direct form-II structure of the system and write the corresponding difference equation.

Soln. For the given system

$$H(z) = \frac{\frac{1}{5}z^{-1} + 1}{\frac{1}{12}z^{-3} + \frac{5}{24}z^{-2} - \frac{1}{4}z^{-1} + 1}$$

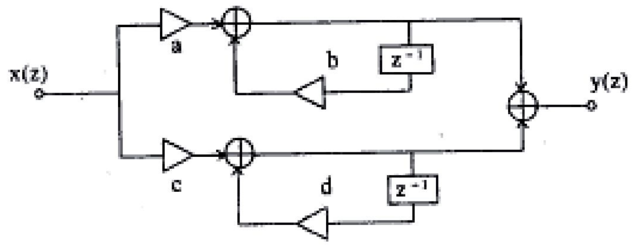
Realization:



5. The transfer function of a causal IIR filter is given by **UPTU 2013-14**

$$H(z) = \frac{5z(3z-2)}{(z+0.5)(2z-1)}$$

Determine the values of multiplier coefficients of the realization structure shown in Figure



Soln.

$$\frac{H(z)}{z} = \frac{2.5(3z-2)}{(z+0.5)(z-0.5)}$$

After partial fraction

$$H(z) = \frac{4.375}{1+0.5z^{-1}} - \frac{1.25}{1-0.5z^{-1}}$$

Therefore a=4.375, c= - 1.25, b= - 0.5, d=0.5

6. how an IIR filter is different than FIR Filter? **UPTU 2015-16, AKTU 2016-17, AKTU 2017-18**

Soln.

a) IIR is infinite and used for applications where linear characteristics are not of concern.
b) FIR filters are Finite IR filters which are required for linear-phase characteristics.
c) IIR is better for lower-order tapping, whereas the FIR filter is used for higher-order tapping.
d) FIR filters are preferred over IIR because they are more stable, and feedback is not involved.
e) IIR filters are recursive and used as an alternate, whereas FIR filters have become too long and cause problems in various applications.

7. For the given system function

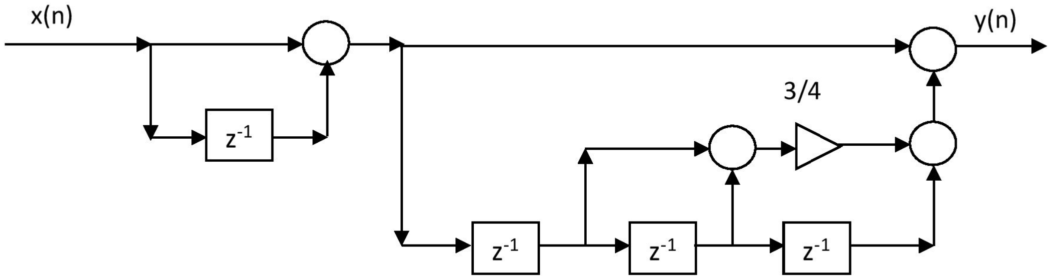
$$H(z) = (1+z^{-1})(1+\tfrac{3}{4}z^{-1}+\tfrac{3}{4}z^{-2}+z^{-3})$$

Obtain cascade realization with minimum number of multipliers. **UPTU 2015-16**

Soln.

$$H(z) = (1 + z^{-1})(1 + \frac{3}{4}(z^{-1} + z^{-2}) + z^{-3})$$

Then

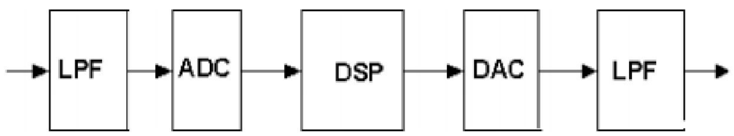


8. Define Digital Signal Processing. **AKTU 2016-17**

Soln. Digital signal processing (DSP) is the process of analyzing and modifying a signal to optimize or improve its efficiency or performance. It involves applying various mathematical and computational algorithms to analog and digital signals to produce a signal that's of higher quality than the original signal.

9. Draw the block diagram of digital signal processing. **AKTU 2016-17**

Soln.



1. An anti-aliasing low-pass filter.
2. An A/D converter whose sampling period T =1/f_s . Here f_s is the sampling frequency with $f_s \geq 2\ f_m$. Here f_m is the highest frequency present in the input signal. Aliasing will occur if the sampling frequency is less than twice the highest frequency contained in the signal.
3. This is the digital signal processor
4. D/A Converter
5. Reconstruction filter

10. Enumerate the Advantages of DSP over ASP. **AKTU 2016-17**

Soln. Digital signal processing has following advantages:

1. Digital signal processing operations can be changed by changing the program in digital programmable system, *i.e.*, this is flexible systems.
2. Better control of accuracy in digital systems compared to analog systems.
3. Digital signals are easily stored on magnetic media such as magnetic tape without loss of quality of reproduction of signal.
4. Digital signals can be processed off line, *i.e.*, these are easily transported.
5. Sophisticated signal processing algorithms can be implemented by DSP method.
6. Digital circuits are less sensitive to tolerances of component values.
7. Digital systems are independent of temperature, ageing and other external parameters.
8. Digital circuits can be reproduced easily in large quantities at comparatively lower cost.
9. Cost of processing per signal in DSP is reduced by time-sharing of given processor among a number of signals.
10. Processor characteristics during processing, as in adaptive filters can be easily adjusted in digital implementation.

11. What are the advantages and disadvantages of digital signal processing **AKTU 2020-21**

12. Distinguish between recursive and non-recursive structure used for the realization of digital system. **AKTU 2020-21, 2021-22**

13. Enlist the Condition for Linear Phase FIR digital filter with 5 Number of samples.

AKTU 2021-22

14. Explain the basic elements required for realization of digital system.

AKTU 2022-23

10 marks Questions:

11. Find the Ladder Structure realization of the system function:

UPTU 2011-12

$$H(z) = \frac{2z^{-2} + 3z^{-1} + 1}{z^{-2} + z^{-1} + 1}$$

Soln: For the given system Routh array can be generated as

z^{-2}	2	3	1
z^{-2}	1	1	1
z^{-1}	1	-1	
z^{-1}	2	1	
1	-(3/2)		
1	1		

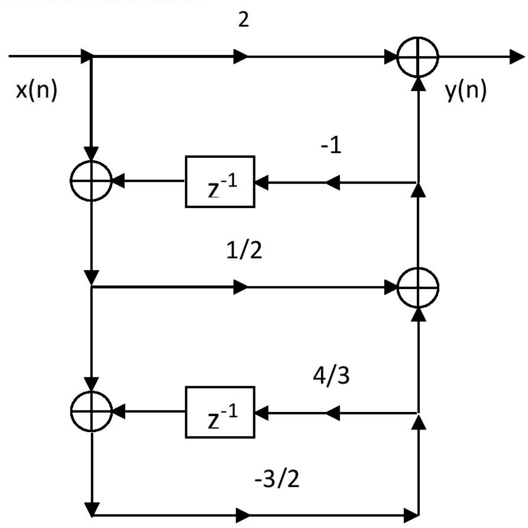
The Ladder structure parameters are

$$\alpha_0 = 2, \beta_1 = 1, \alpha_1 = \frac{1}{2}, \beta_2 = -\frac{4}{3}, \alpha_2 = -\frac{3}{2}$$

and the continued fraction expansion is

$$H(z) = 2 + \cfrac{1}{z^{-1} + \cfrac{1}{\frac{1}{2} + \cfrac{1}{-\frac{4}{3}z^{-1} + \cfrac{1}{-\frac{3}{2}}}}}$$

Then the Ladder realization is shown below



11. System function $H(z) = \frac{Y(z)}{X(z)}$ for a linear shift invariant system is given by

$$H(z) = \frac{2}{z^{-3} + 4z^{-2} + z^{-1} + 2}.$$

12. Find the two port realization of the system.

UPTU 2011-12

Soln: For the given system Routh array can be generated as

z^{-3}	1	1
z^{-2}	4	2
z^{-1}	1/2	
1	2	

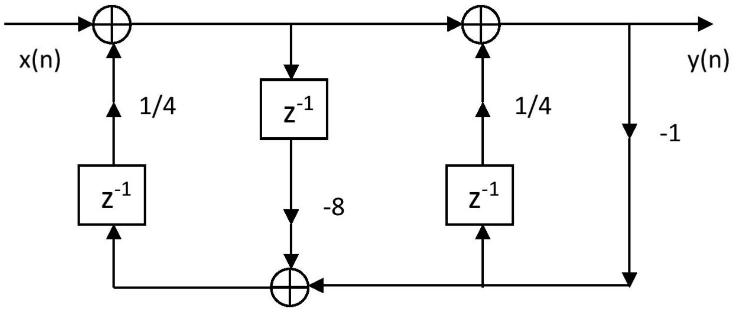
The Ladder structure parameters are

$$\alpha_0 = \frac{1}{4}, \alpha_1 = 8, \alpha_2 = \frac{1}{4}$$

and the continued fraction expansion is

$$H(z) = \frac{1}{4}z^{-1} + \cfrac{1}{8z^{-1} + \cfrac{1}{\frac{1}{4}z^{-1}}}$$

Then the Ladder realization is shown below



13. Find the Parallel form realization for a discrete time linear, causal system given by the difference equation: Y(n)= 3/4 y(n-1)-1/8 y(n-2)+ x(n)+ 1/3 x(n-1).

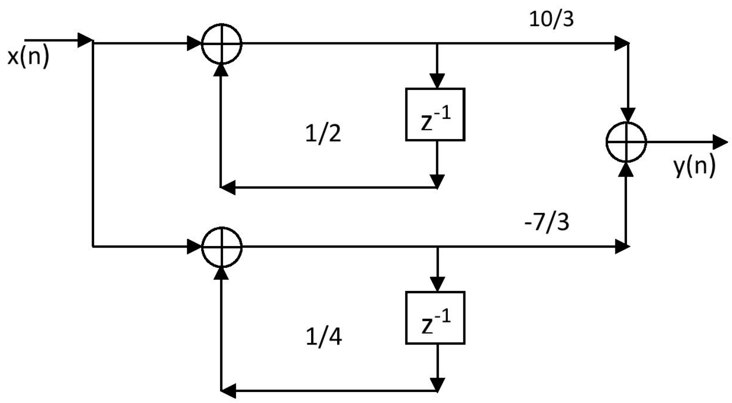
UPTU 2012-13

Soln. For the given system

$$H(z) = \frac{1 + \frac{1}{3}z^{-1}}{\frac{1}{8}z^{-2} - \frac{3}{4}z^{-1} + 1}$$

For parallel form realization H(z) is converted into its partial fraction form as

$$H(z) = \frac{\frac{10}{3}}{-\frac{1}{2}z^{-1}+1} + \frac{-\frac{7}{3}}{-\frac{1}{4}z^{-1}+1}$$



14. Obtain the Ladder structure for

$$H(z) = \frac{1}{z^{-3}+2z^{-2}+2z^{-1}+1}.$$

Soln. For the given system Routh array can be generated as

z^{-3}	1	2
z^{-2}	2	1
z^{-1}	3/2	
1	1	

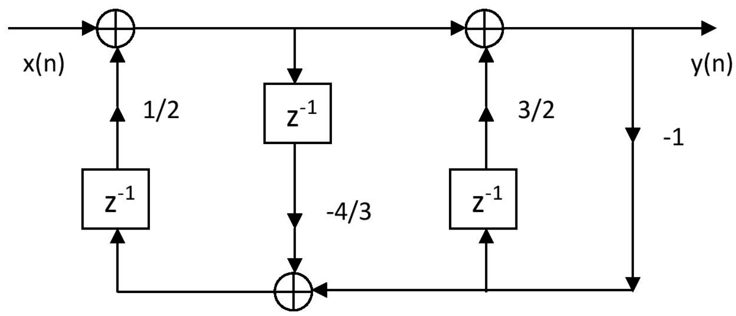
The Ladder structure parameters are

$$\alpha_0 = \frac{1}{2}, \alpha_1 = \frac{4}{3}, \alpha_2 = \frac{3}{2}$$

and the continued fraction expansion is

$$H(z) = \frac{1}{2}z^{-1} + \frac{1}{\frac{4}{3}z^{-1} + \frac{1}{\frac{3}{2}z^{-1}}}$$

Then the Ladder realization is shown below



15. Sketch the Ladder structure for the system: **UPTU 2013-14**

$$H(z) = \frac{(1 - 0.6z^{-1} + 1.2z^{-2})}{(1 + 0.15z^{-1} - 0.64z^{-2})}$$

Soln. For the given system Routh array can be generated as

z^{-2}	1.2	- 0.6	1
z^{-2}	- 0.64	0.15	1
z^{-1}	- 0.32	2.88	
z^{-1}	- 5.61	1	
1	2.82		
1	1		

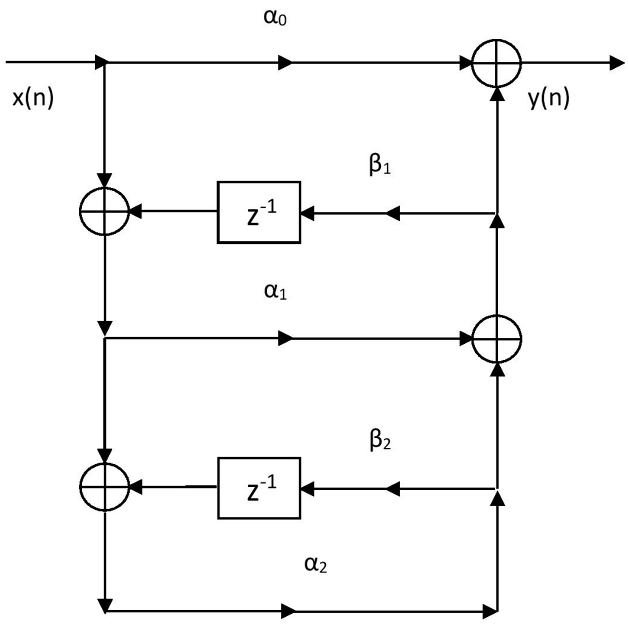
The Ladder structure parameters are

$$\alpha_0 = -1.875, \beta_1 = 2, \alpha_1 = 0.057, \beta_2 = -1.99, \alpha_2 = 2.82$$

and the continued fraction expansion is

$$H(z) = -1.875 + \cfrac{1}{2z^{-1} + \cfrac{1}{0.057 + \cfrac{1}{-1.99z^{-1} + \cfrac{1}{2.82}}}}$$

Then the Ladder realization is shown below



16. Obtain Direct form- I and Direct form - II and Cascade structure for following system :

$$y(n) = [-0.1y(n-1) - 0.72y(n-2) + 0.7x(n) - 0.2x(n-2)]$$

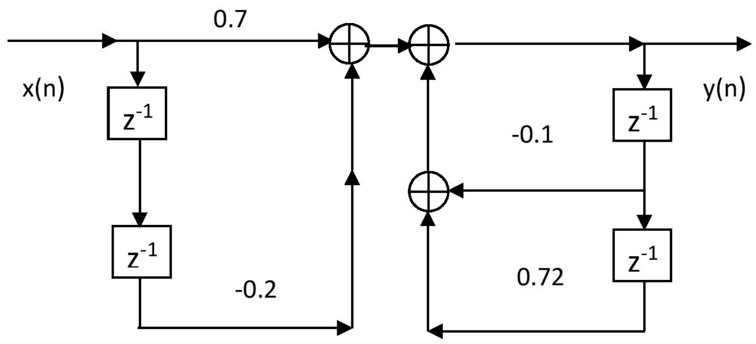
UPTU 2013-14

Soln. For the given system

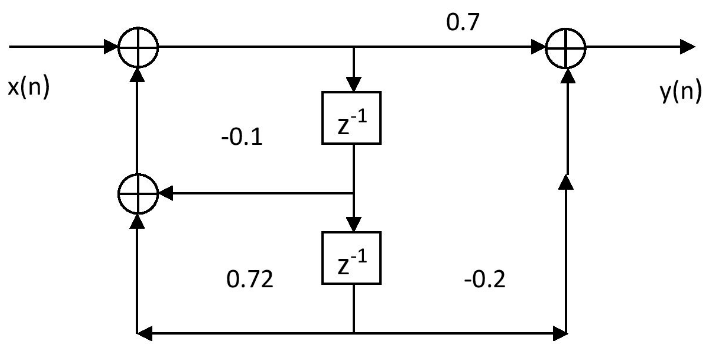
$$H(z) = \frac{-0.2z^{-2} + 0.7}{0.72z^{-2} + 0.1z^{-1} + 1}$$

Realization:

Direct form-I/ Cascade Form



Direct form-II



17. Discuss the importance of realization of digital system. Find the system function of the system shown in following figure. **UPTU 2014-15**

Soln. The block diagram construction of the filter is called a realization of the filter. Realization of a filter at the block diagram level is essentially a flow graph of the signals in the filter. It includes operations such as delays, additions and multiplications of signals by constant coefficients. It ignores ordering of operations, accuracy, scaling, and the like. A given filter can be realized in infinitely many ways. Different realizations differ in their properties and some are better than others. If the system is to be implemented the block diagram can be converted into a program that runs on a digital computer. Alternatively, the structure in block diagram form implies a hardware configuration for implementing the system
From the Mason Gain formula the transfer function for the given system is

$$H(z) = \frac{(1-b_3)\{(1-b_3)z^{-2} + b_2z^{-1} - b_3\}}{b_3z^{-2} - b_2z^{-1} + 1}$$

18. Discuss the advantages of ladder form of realization. Find the ladder form of realization of the system function given by: **UPTU 2014-15**

$$H(z) = \frac{2}{z^{-3}+4z^{-2}+z^{-1}+2}.$$

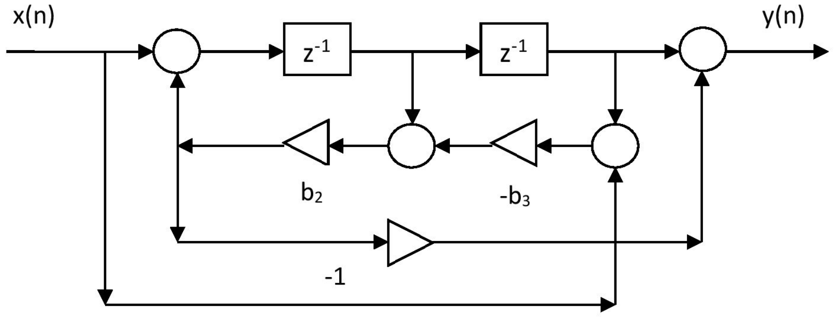
Soln.

Ladder realization provides desirable coefficient-sensitivity property to the system. This property implies that small changes in the filter parameters have little effect on its performance.

For the given system Routh array can be generated as

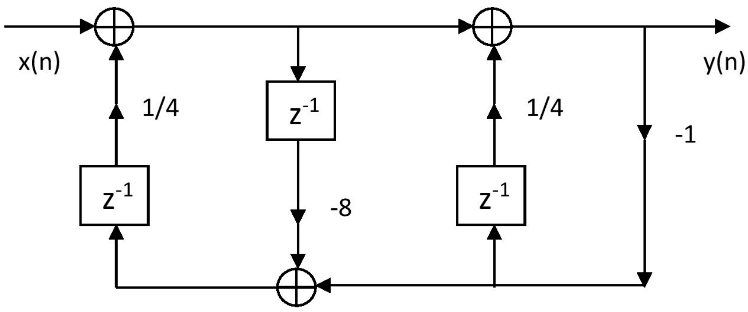
z^{-3}	1	1
z^{-2}	4	2
z^{-1}	1/2	
1	2	

The Ladder structure parameters are $\alpha_0 = \frac{1}{4}, \alpha_1 = 8, \alpha_2 = \frac{1}{4}$
and the continued fraction expansion is



$$H(z) = \frac{1}{4}z^{-1} + \cfrac{1}{8z^{-1} + \cfrac{1}{\frac{1}{4}z^{-1}}}$$

Then the Ladder realization is shown below



19. Obtain the Parallel form realization for the transfer function H(z) given below:

AKTU 2016-17

$$H(z) = \frac{2 + z^{-1} + \frac{1}{4}z^{-2}}{(1 + \frac{1}{2}z^{-1})(1 + z^{-1} + \frac{1}{2}z^{-2})}$$

Soln. After partial fraction

$$H(z) = \frac{1}{1 + \frac{1}{2}z^{-1}} + \frac{1 - \frac{1}{2}z^{-1}}{1 + 2z^{-1} + \frac{1}{2}z^{-2}}$$

Now realize h(z) in parallel form same as Q3

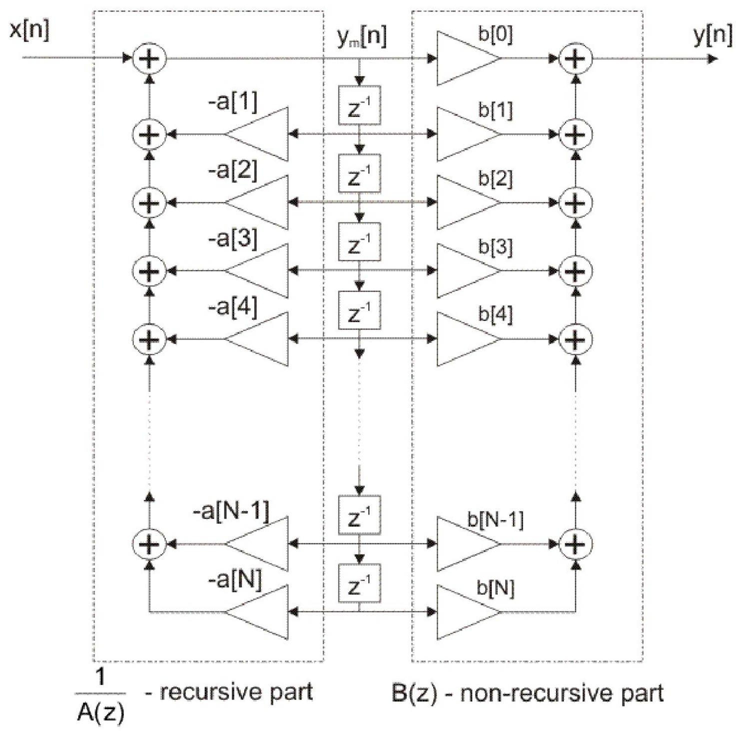
15 marks Questions:

20 a. What do you mean by canonical form of realization? **UPTU 2011-12**

Soln: Direct Canonical Realization

Direct canonical realization structure has reduced number of delay lines to the minimum, that is, N delay lines. This way, one of the main disadvantages of direct and direct transpose realization structures is eliminated. Recursive and non-recursive parts of IIR filter are not considered separately, which causes implementation to be more complex than for direct realization structure. A good thing is that the coefficients are the same as for direct realization. Direct canonic structure uses N delay elements, (2N+1) multipilications and 2N additions.

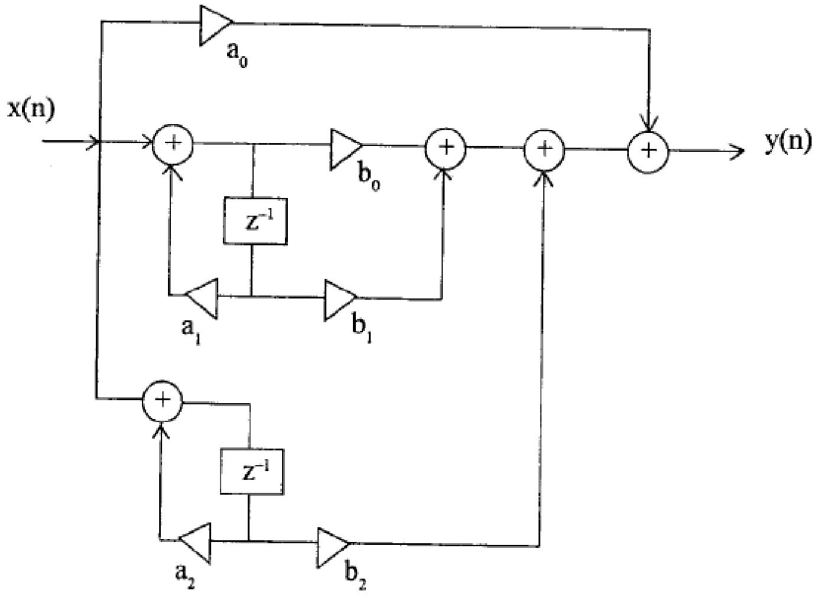
Figure illustrates the block diagram describing direct canonic realization structure of IIR filter.



Direct canonic realization structure block diagram

b. Determine the system function H(z) for the following system shown in figure

UPTU 2011-12



Soln.

$$H(z) = \frac{Y(z)}{X(z)} = a_0 + \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} + \frac{b_2 z^{-1}}{1 - a_2 z^{-1}}$$

21. Explain direct form-I and Direct form-II for realization of IIR filters.

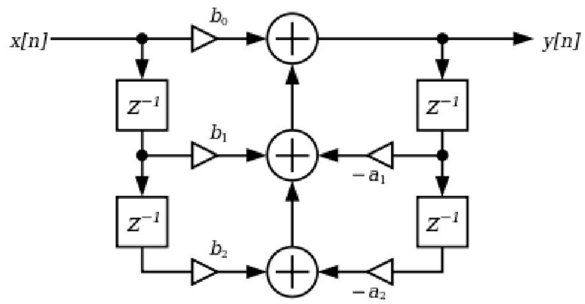
Soln: Filter realization

After a filter is designed, it must be *realized* by developing a signal flow diagram that describes the filter in terms of operations on sample sequences.

A given transfer function may be realized in many ways. Consider how a simple expression such as $ax + bx + c$ could be evaluated – one could also compute the equivalent $x(a + b) + c$. In the same way, all realizations may be seen as "factorizations" of the same transfer function, but different realizations will have different numerical properties. Specifically, some realizations are more efficient in terms of the number of operations or storage elements required for their implementation, and others provide advantages such as improved numerical stability and reduced round-off error. Some structures are better for fixed-point arithmetic and others may be better for floating-point arithmetic.

Direct Form I

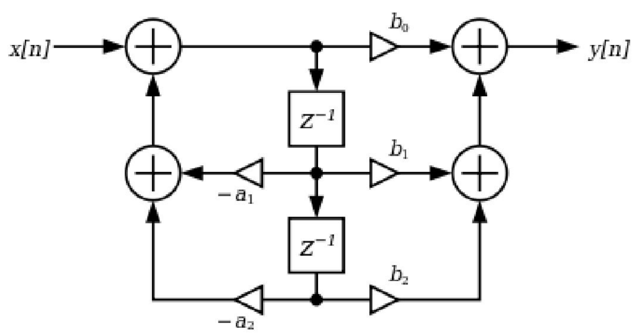
A straightforward approach for IIR filter realization is Direct Form I, where the difference equation is evaluated directly. This form is practical for small filters, but may be inefficient and impractical (numerically unstable) for complex designs. In general, this form requires 2N delay elements (for both input and output signals) for a filter of order N.



Direct Form II

The alternate Direct Form II only needs N delay units, where N is the order of the filter – potentially half as much as Direct Form I. This structure is obtained by reversing the order of the numerator and denominator sections of Direct Form I, since they are in fact two linear systems, and the commutativity property applies. Then, one will notice that there are two columns of delays (z^{-1}) that tap off the center net, and these can be combined since they are redundant, yielding the implementation as shown below.

The disadvantage is that Direct Form II increases the possibility of arithmetic overflow for filters of high Q or resonance. It has been shown that as Q increases, the round-off noise of both direct form topologies increases without bounds. This is because, conceptually, the signal is first passed through an all-pole filter (which normally boosts gain at the resonant frequencies) before the result of that is saturated, then passed through an all-zero filter (which often attenuates much of what the all-pole half amplifies).



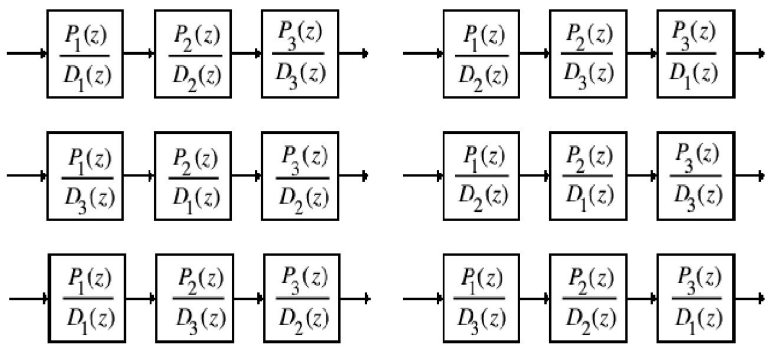
22. Explain Parallel and cascade form realizations for IIR filters.

Soln: Cascade Form: By expressing the numerator and the denominator polynomials of the transfer function as a product of polynomials of lower degree, a digital filter can be realized as a cascade of low-order filter sections

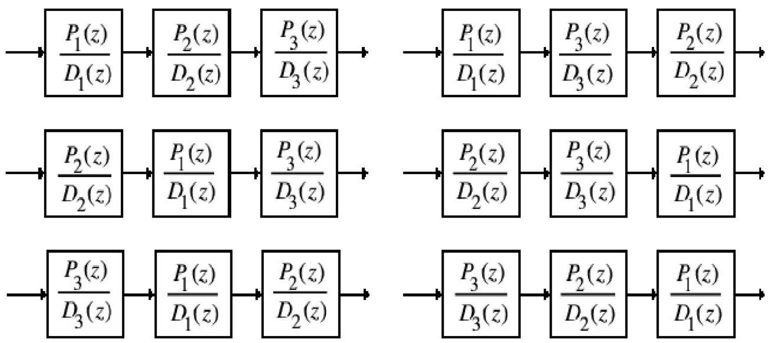
Consider, for example, $H(z) = P(z)/D(z)$ expressed as

$$H(z) = \frac{P(z)}{D(z)} = \frac{P_1(z)P_2(z)P_2(z)}{D_1(z)D_2(z)D_3(z)}$$

Examples of cascade realizations obtained by different pole-zero pairings are shown below



Examples of cascade realizations obtained by different ordering of sections are shown below.



There are altogether a total of 36 different cascade realizations of

$$H(z) = \frac{P_1(z)P_2(z)P_2(z)}{D_1(z)D_2(z)D_3(z)}$$

based on pole-zero-pairings and ordering

Due to finite wordlength effects, each such cascade realization behaves differently from others

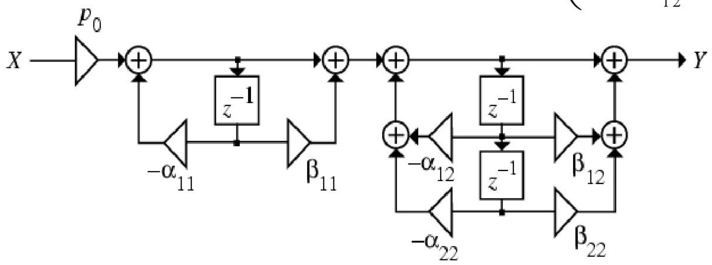
Usually, the polynomials are factored into a product of 1st-order and 2nd-order polynomials:

$$H(z) = p_0 \prod_k \left(\frac{1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}}{1 + \alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}} \right)$$

In the above, for a first-order factor $\alpha_{2k} = \beta_{2k} = 0$

Consider the 3rd-order transfer function $H(z) = p_0 \left(\frac{1+\beta_{11} z^{-1}}{1+\alpha_{11} z^{-1}} \right) \left(\frac{1+\beta_{12} z^{-1} + \beta_{22} z^{-2}}{1+\alpha_{12} z^{-1} + \alpha_{22} z^{-2}} \right)$

One possible realization is shown below



Parallel Form: A partial-fraction expansion of the transfer function in z^{-1} leads to the **parallel form I** structure. Assuming simple poles, the transfer function $H(z)$ can be expressed as

$$H(z) = \gamma_0 + \sum_k \left(\frac{\gamma_{0k} + \gamma_{1k} z^{-1}}{1+\alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}} \right)$$

In the above for a real pole $\alpha_{2k} = \gamma_{1k} = 0$

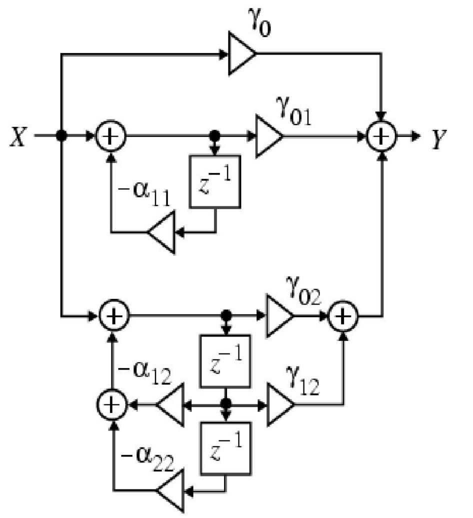
A direct partial-fraction expansion of the transfer function in z leads to the **parallel form II** structure

Assuming simple poles, the transfer function $H(z)$ can be expressed as

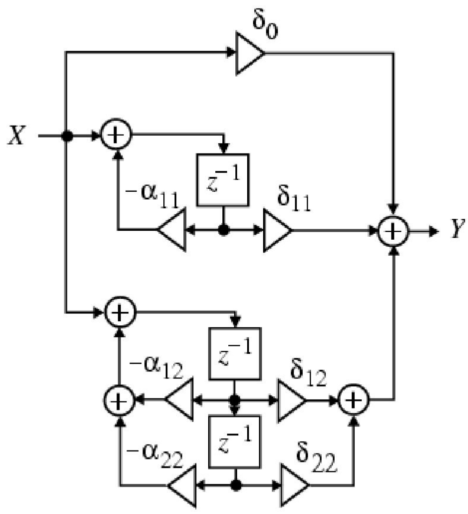
$$H(z) = \delta_0 + \sum_k \left(\frac{\delta_{0k} z^{-1} + \delta_{2k} z^{-2}}{1+\alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}} \right)$$

In the above for a real pole $\alpha_{2k} = \delta_{2k} = 0$

The two basic parallel realizations of a 3rd-order IIR transfer function are shown below



Parallel form I



Parallel form II

23. Obtain Direct form- I and Direct form - II and Parallel form structures for the following system :

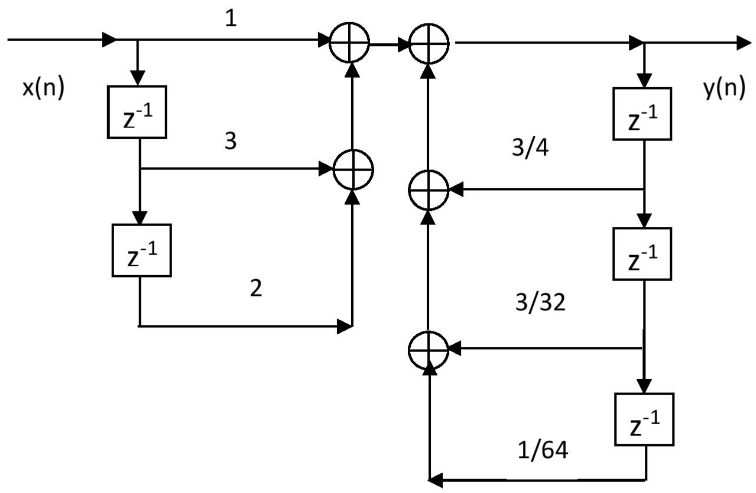
$$y(h) = [\frac{3}{4}y(h-1) + \frac{3}{32}y(h-2) + \frac{1}{64}y(h-3) + x(h) + 3x(h-1) + 2x(h-2)]$$

UPTU 2015-16

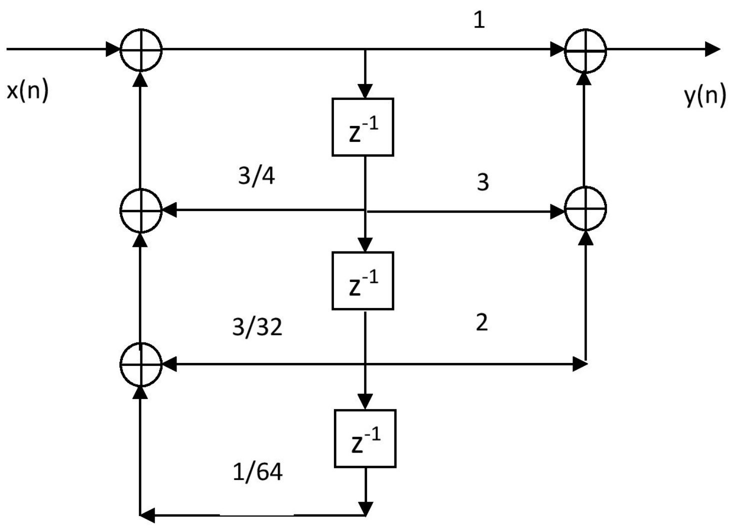
Soln. For the given system

$$H(z) = \frac{2z^{-2} + 3z^{-1} + 1}{-\frac{1}{64}z^{-3} - \frac{3}{32}z^{-2} - \frac{3}{4}z^{-1} + 1}$$

Realization:
Direct form-I

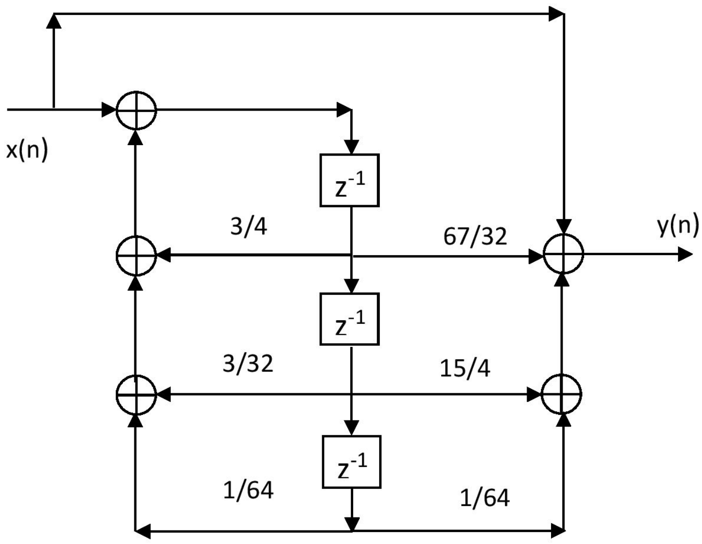


Direct form-II



Parallel Form
After division

$$H(z) = 1 + \frac{\frac{1}{64}z^{-3} + \frac{15}{4}z^{-2} + \frac{67}{32}z^{-1}}{-\frac{1}{64}z^{-3} - \frac{3}{32}z^{-2} - \frac{3}{4}z^{-1} + 1}$$



24. Consider the causal linear-shift-invarient fiter with the system function

UPTU 2015-16

$$H(z) = \frac{1 + 0.875z^{-1}}{(1 + 0.2z^{-1} + 0.9z^{-2})(1 - 0.7z^{-1})}$$

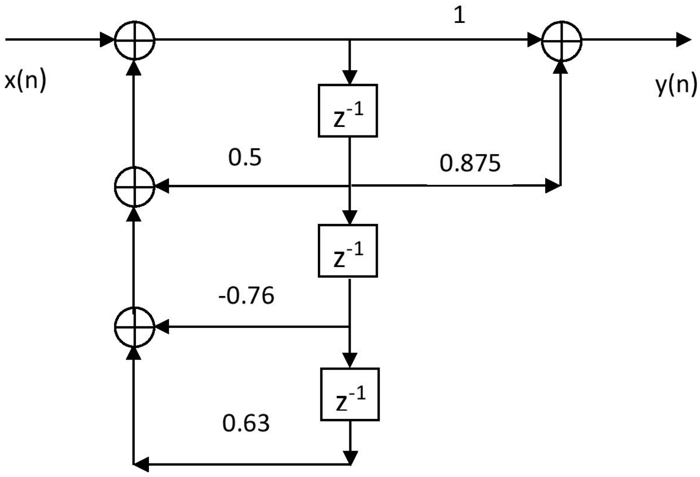
Obtain following realizations.

a) Direct Form II
b) A cascade of first order and secnd order system realized in transposed DF II.
c) A parallel connection of first order and second order systems realized in DF II.

Soln.

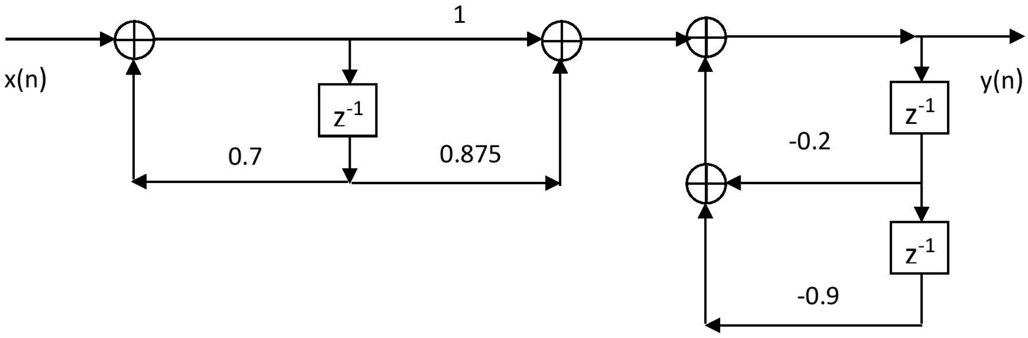
a) Direct Form II

$$H(z) = \frac{1 + 0.875z^{-1}}{(1 - 0.5z^{-1} + 0.76z^{-2} - 0.63z^{-3})}$$

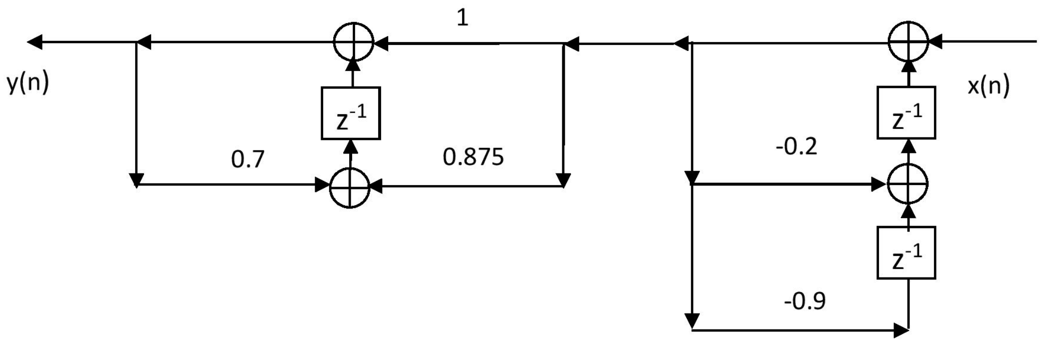


b) Cascade Form

$$H(z) = \frac{1 + 0.875z^{-1}}{(1 - 0.7z^{-1})} \cdot \frac{1}{(1 + 0.2z^{-1} + 0.9z^{-2})}$$



Cascade in transposed DF II

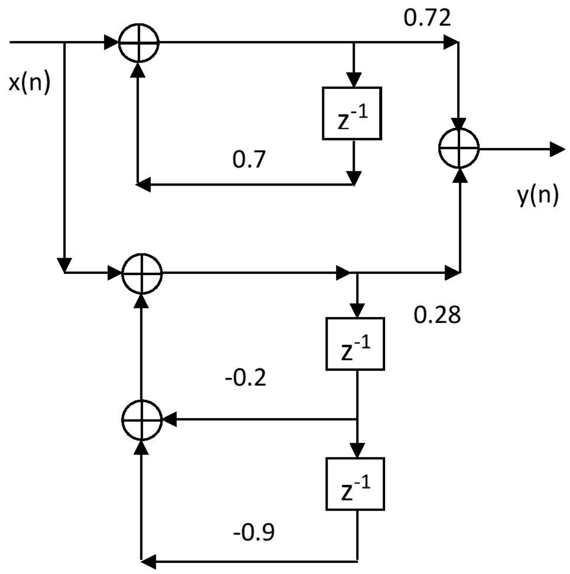


c) Parallel form

$$\frac{H(z)}{z} = \frac{z^2 + 0.875z}{(z^2 + 0.2z + 0.9)(z - 0.7)}$$

After partial fraction

$$H(z) = \frac{0.72}{(1 - 0.7z^{-1})} + \frac{0.28}{(1 + 0.2z^{-1} + 0.9z^{-2})}$$



25. Obtain the ladder structure for the system function H(z) given below.

AKTU 2016-17, 2017-18

$$H(z) = \frac{2 + 8z^{-1} + 6z^{-2}}{1 + 8z^{-1} + 12z^{-2}}$$

Soln. For the given system Routh array can be generated as

z^{-2}	6	8	2
z^{-2}	12	8	1
z^{-1}	4	3/2	
z^{-1}	7/2	1	
1	15/14		
1	1		

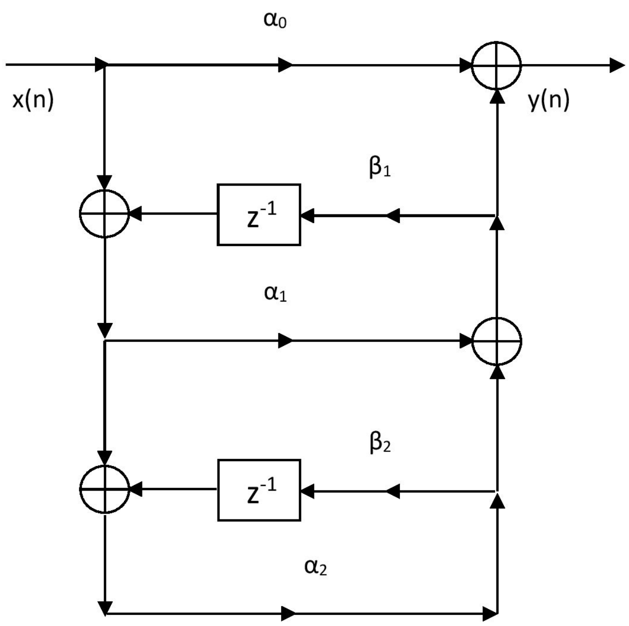
The Ladder structure parameters are

$$\alpha_0 = 1/2, \beta_1 = 3, \alpha_1 = 8/7, \beta_2 = 49/15, \alpha_2 = 15/14$$

and the continued fraction expansion is

$$H(z) = 1/2 + \cfrac{1}{3z^{-1} + \cfrac{1}{8/7 + \cfrac{1}{(\frac{49}{15})z^{-1} + \cfrac{1}{(\frac{15}{14})}}}}$$

Then the Ladder realization is shown below



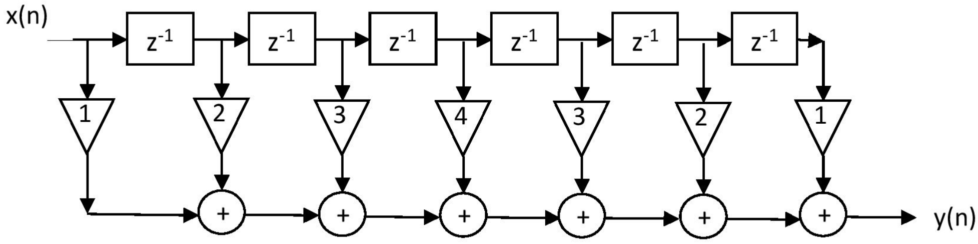
Additional questions

1. Determine the direct form realization of the linear phase filter given by

UPTU 2012-13

$$h(n) = \{1, 2, 3, 4, 3, 2, 1\}$$

Ans. Direct form realization for the linear phase filter



2. Consider an FIR filter described by the system function

UPTU 2012-13

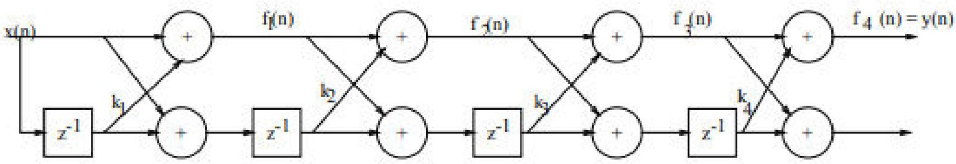
$$H(z) = 1 + 2.88z^{-1} + 3.4048z^{-2} + 1.74z^{-3} + 0.4z^{-4}$$

i) Sketch the lattice realization of the filter.

ii) Is the system minimum phase

Ans.

A4(z) = H(z) = 1 + 2.88z−1 + 3.4048z−2 + 1.74z−3 + 0.4z−4
B4(z) = 0.4 + 1.74z−1 + 3.4048z−2 + 2.88z−3 + z−4
Hence, K4 = 0.4
A3(z) = (A4(z) − k4B4(z))/(1 − k24) = 1 + 2.6z−1 + 2.432z−2 + 0.7z−3
B3(z) = 0.7 + 2.432z−1 + 2.6z−2 + z−3
Hence, K3 = 0.7
A2(z) = (A3(z) − k3B3(z))/(1 − k23) = 1 + 1.76z−1 + 1.2z−2
B2(z) = 1.2 + 1.76z−1 + z−2
Then, K2 = 1.2
A1(z) = (A2(z) − k2B2(z))/(1 − k22) = 1 + 0.8z−1
Therefore, K1 = 0.8
Since K2 > 1, the system is not minimum phase.



Lattice structure

3. Obtain the direct form I, direct form II, cascade and parallel form realization for the following system:

AKTU 2017-18

y (n) = - 0.1 y(n-1) + 0.2 y(n-2) +3 x(n) + 3.6 x(n-1) + 0.6x(n-2)

Ans. Same as Q16

4. Determine the coefficients of a continued-fraction expansion of H (z); Also draw ladder realization structure of H(z).

AKTU 2020-21

$$H(z) = \frac{2 + 8z^{-1} + 6z^{-2}}{(1 + 8z^{-1} + 12z^{-2})}$$

5. Obtain the direct form-I, direct form-II, cascade, and parallel form realization of a given LTI system: y(n)=-0.1y(n-1) +0.72y(n-2) +0.7x(n)-0.25x(n-2)

AKTU 2020-21

6. For $H(z) = 1 + 2z^{-1} - z^{-2} + 3z^{-3} + 3z^{-4} - z^{-5} + 2z^{-6} + z^{-7}$ Draw the direct form and linear form FIR implementation. Also compare the implementation.

AKTU 2020-21

7. Realize the given H(z) for using ladder structure. **AKTU 2021-22**

$$H(z)=\frac{2+ 8z^{-1} + 6z^{-2}}{1+ 8z^{-1} + 12z^{-2}}$$

8. Describe the linear phase FIR systems, &For h (n) = [1/2, 1/3, 1/5, 1/3, 1/2] Realize H(z) of the Linear Phase FIR system for the given impulse response.

AKTU 2021-22

9. Find out the direct form-I & direct form-II realization of a discrete-time system represented by the transfer function

AKTU 2021-22

$$y(n) = -\frac{13}{12} y(n-1) - \frac{9}{24} y(n-2) - \frac{1}{24} y(n-3) + x(n) + 4\, x(n-1) + 3\, x(n-2)$$

10. Determine DF – I & DF – II realization for a following IIR transfer function

AKTU 2022-23

$H(z) = (0.28z^2 + 0.319z + 0.04)/(0.5z^3 + 0.3z^2 + 0.17z - 0.2)$

11. Obtain direct form and cascade form realization for the transfer function of a FIR system given by $H(z) = \left(1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2}\right)\left(1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2}\right)$

AKTU 2022-23

12. (i) Explain the technologies used for DSP in detail. **AKTU 2022-23**

(ii) Compare IIR and FIR digital filters.

UNIT-2

(Infinite Impulse Response Digital (IIR) Filter Design)

Section-A- Two marks questions

1. How one can design Digital Filter from Analog Filter? **UPTU 2010-11**

Ans. Digital filters can be designed using analog design methods by following these steps:

1. Filter specifications are specified in the digital domain. The filter type (highpasslowpass,bandpass etc.) is specified.
2. An equivalent lowpass filter is designed that meets these specifications.
3. The analog lowpass filter is transformed using spectral transformations into the correct type of filter.
4. The analog filter is transformed into a digital filter using a particular mapping.

There are many different types of spectral transformations and there are many mappings from analog to digital filters. the most famous mapping is known as the bilinear transform

2.Prove that physically realizable IIR filter can not have linear phase. **UPTU 2010-11**

Ans. For digital filters, linear phase places the following requirement on the transfer function:

$$H(z)=H(z^{-1})$$

That restriction implies a linear phase IIR filter would need to have poles both inside and outside the unit circle, making it unstable. As a result, a causa and stable IIR filter cannot have a linear phase.

3.Given the specification α_p = 1db, α_s = 40db, Ω_p = 200 rad/sec, Ω_s = 600 rad/sec.Determine the order of the butterworth filter where Ω_p and Ω_s are the passband and stopband frequencies and α_p and α_s are passband and stopband attenuation.

UPTU 2010-11

Ans. Given

α_p = 1db,

α_s = 40db,

Ω_p = 200 rad/sec,

Ω_s = 600 rad/sec

Order of the Butterworth filter

$$N \geq \frac{log_{10}\left(\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}\right)}{log_{10}\left(\frac{\Omega_s}{\Omega_p}\right)} \geq \frac{log_{10}\left(\frac{10^4-1}{10^{0.1}-1}\right)}{log_{10}\left(\frac{600}{200}\right)} \geq \frac{log_{10}\left(\frac{10^4-1}{10^{0.1}-1}\right)}{log_{10}\left(\frac{600}{200}\right)} \geq 9.6$$

Therefore $N \cong 10$.

4.What are the advantages and disadvantages of Bilinear Transformation?**UPTU 2010-11**

Ans. Advantages:

- The bilinear transformation provides one-to-one mapping.
- Stable continuous systems can be mapped into realizable, stable digital systems.
- There is no aliasing.

Disadvantage:

- The mapping is highly non-linear producing frequency, compression at high frequencies.
- Neither the impulse response nor the phase response of the analog filter is preserved in a digital filter obtained by bilinear transformation.

5. Give any three properties of Butterworth low pass filter. **UPTU 2010-11**

Ans.

- Very close to the ideal near ω=0 and ω=∞,
- Very smooth at all frequencies with a monotonic decrease from ω=0 to ∞.
- Largest difference between the ideal and actual responses near the transition at ω=1 where $|F(j1)|^2=1/2$.
- The response is called maximally-flat at zero and infinity
- This filter has a very smooth frequency response and, although not explicitly designed for, has a smooth phase response.

6. Convert the analog filter with the system function

$$H_a(s) = \frac{s+0.2}{(s+0.2)^2+9}$$

into a digital IIR filter using impulse invariant technique. Assume T = ls.

UPTU 2012-13

Ans. By using the impulse invariant transformation property

$$\frac{s+a}{(s+a)^2+b^2} \rightarrow \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

We get

a = 0.2 and b = 3, for T= 1s.

$$H(z) = \frac{1-e^{-0.2}(\cos 3)z^{-1}}{1-2e^{-0.2}(\cos 3)z^{-1}+e^{-0.4}z^{-2}}$$

7. Explain the term group delay with respect to filters. **UPTU 2012-13**

Ans. In signal processing, group delay is a measure of the time delay of the amplitude envelopes of the various sinusoidal components of a signal through a device under test, and is a function of frequency for each component. Phase delay, in contrast, is a measure of the time delay of the phase as opposed to the time delay of the amplitude envelope.

8. Discuss the properties of Chebyshev Polynomial. **UPTU 2012-13**

Ans.

i. The Chebyshev polynomials of the first kind are defined through the identity

$$T_N(x) = \cos\left(N\cos^{-1}(x)\right) \quad ; \quad |x| \le 1$$
$$T_N(x) = \cosh\left(N\cosh^{-1}(x)\right) \quad ; \quad |x| > 1$$

ii. Chebyshev polynomials can be generated in the following way.
Set $T_0(x) = 1$ and $T_1(x) = x$ and use the recurrence relation
$T_N(x) = 2xT_{N-1}(x) - T_{N-2}(x)$ for $N = 2, 3, \ldots$.

iii. The coefficient of x^N in $T_N(x)$ is 2^{N-1} when $N \ge 1$.

iv. When N = 2M, $T_{2M}(x)$ is an even function, that is,
$T_{2M}(-x) = T_{2M}(x)$.

v. When N = 2M + 1, $T_{2M+1}(x)$ is an odd function, that is,
$T_{2M+1}(-x) = -T_{2M+1}(x)$.

vi. $|T_N(x)| \le 1$ for $-1 \le x \le 1$.

9. State the condition for design of stable digital filter from stable analog filter.

UPTU 2012-13

Ans. The stability of a filter is related to the location of the poles. For a stable analog filter the poles should lie on the left half of s-plane. For a stable digital filter the poles should lie inside the unit circle in the z-plane.

10. Convert analog filter to digital filter whose system function is

$$H_a(s) = \frac{36}{(s+0.1)^2 + 36}$$

The digital filter should have a resonant frequency of $\omega_r = 0.2\pi$. Use Bilinear Transformation Technique. **UPTU 2013-14**

Ans. For the given system $\Omega_c = 6$.

$$\Omega_c = \frac{2}{T}\tan\left(\frac{\omega_r}{2}\right)$$

$$T = \frac{2}{\Omega_c}\tan\left(\frac{\omega_r}{2}\right) = \frac{2}{6}\tan\left(\frac{0.2\pi}{2}\right) = 0.108s$$

By using bilinear transformation

$$H(z) = H_a(s)\Big|_{s=\frac{2(1-z^{-1})}{T(1+z^{-1})}}$$

$$H(z) = \frac{36T^2(1+z^{-1})^2}{((2+0.1T)-(2-0.1T)z^{-1})^2 + 36T^2(1+z^{-1})^2}$$

11. An analog filter has the system function$H(s) = \frac{1}{(s+0.1)^2+9}$. Convert this into a digital filter using impulse invariance method. Assume T=1s.

Ans. From the direct conversion formula

$$H(s) = \frac{1}{(s+0.1)^2+9} = \frac{3/3}{(s+0.1)^2+3^2}$$

Is converted to H(z) as

$$H(z) = \frac{e^{-0.1}\sin(3)z^{-1}}{1 - 2e^{-0.1}\cos(3)z^{-1} + e^{-0.2}z^{-2}}$$

Section-B- 10 marks questions

12. Why is Frequency transformation needed ? What are the different types of frequency transformations? **UPTU 2013-14, AKTU 2017-18**

Ans. There are basically four types of frequency selective filters, low-pass, high-pass, band-pass and band-reject. The ideal responses of these filters are not obtainable in practice because of sharp corners.

Many filters are designed by first designing a low-pass filter, the low-pass filter can be considered as a prototype filter and its system function can be obtained, then by using a frequency transformation it can be converted to obtain a desired filter.

There are mainly two types of frequency transformations:

1. Analog Frequency Transformation and
2. Digital Frequency Transformation

Analog Frequency Transformation

The frequency transformations that can be used to obtain a low-pass, high-pass, band-pass, or band-reject analog filter from a normalized low-pass analog filter are shown in table below:

Filter Type	Transformation
Low-pass	$S = s/\Omega_c$
High-pass	$S = \Omega_c/s$
Band-pass	$S = \frac{Q(s^2 + {\Omega_0}^2)}{\Omega_0 s}$
Band-reject	$S = \frac{\Omega_0 s}{Q(s^2 + {\Omega_0}^2)}$

For low pass and high pass filters, Ω_c is the cutoff frequency. For band-pass and band-reject filters, Ω_0 is the center frequency and Q is the quality factor given by

$${\Omega_0}^2 = \Omega_1\Omega_2$$

$$Q = \frac{\Omega_0}{(\Omega_2 - \Omega_1)}$$

Where Ω_1 and Ω_2 are the lower and upper cutoff frequencies, respectively. The quantity $\Omega_2 - \Omega_1$ is the bandwidth.

Digital Frequency Transformation

Frequency transformations are available also for transforming a low pass digital filter into another low pass, high pass, band pass or band reject digital filters.

13. Determine the order and poles of low pass Butterworth filter having 3dB attenuation at 500 Hz and attenuation of 40 dB at 1000 Hz by using impulse invariance transformation.

UPTU 2010-11

Ans. The critical frequencies are

– 3 dB frequency ω_1 and the stopband frequency ω_2, which are

$\omega_1 = 1000\pi = \Omega_1$ [when IIT is used for T=1]

$\omega_2 = 2000\pi = \Omega_2$

For an attenuation of $A_2 = 40$ dB, & $A_1 = 3$ dB We obtain

$N = \log_{10}((10^4 - 1)/ (10^{0.3} - 1))/2\log_{10}2 = 6.64$

To meet the desired specifications, we select N = 7.

$\Omega_c = 1000\pi/(10^{0.3} - 1)^{1/14} = 3142.66$

The pole positions are

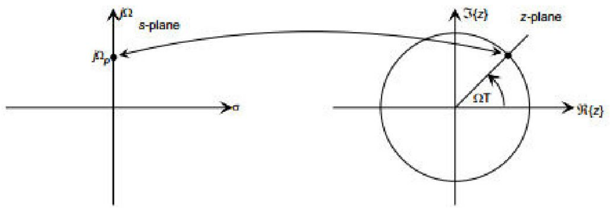
$s_k = 3142.66\ e^{j[\pi/2 + (2k+1)\pi/14]}$, k = 0, 1, 2, …, 6.

14. Describe the complete mapping with expressions and diagrams from s-plane into z-plane if bilinear transformation is used.

Ans. The bilinear transform method uses the truncated series approximation $s \to \frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})}$

With this transformation the digital filter is designed from the prototype using $H(z) = H_a(s)|_{s\to\frac{2(1-z^{-1})}{T(1+z^{-1})}}$

The bilinear transform maps the left half s-plane to the interior of the unit circle, and thus preserves stability. In addition, we will see below that it maps the entire imaginary axis of the s-plane to the unit circle, and thus avoids aliasing in the frequency response.

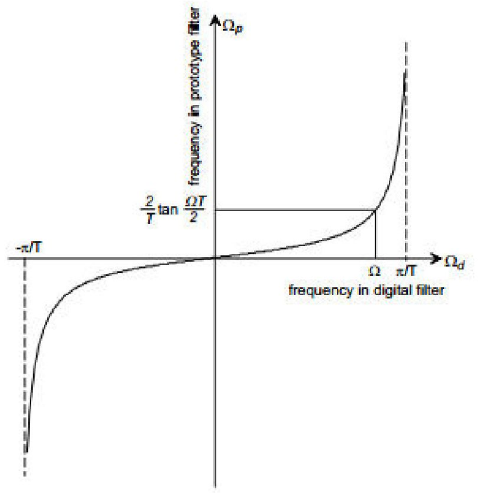


Thus every point on the frequency response of the continuous-time prototype filter, is mapped to a corresponding point in the frequency response of the discrete-time filter, although with a different frequency. This means that every feature in the frequency response of the prototype filter is preserved, with identical gain and phase shift, at some frequency the digital filter.

The mapping $s \to \frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})}$ implies that when z = e^{sT}

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

and there is no aliasing in the frequency response.



15. Design a digital Chebyshev filter from the specification given below:**AKTU 2016-17**

$$0.77 \leq |H(e^{j\omega})| \leq 1 \qquad 0 \leq \omega \leq 0.2\pi$$

$$|H(e^{j\omega})| \leq 0.1 \qquad 0.5\pi \leq \omega \leq \pi$$

Using Bilinear Transformation with T=1s.

Ans. Given

$A_1 = 0.77$

$A_2 = 0.1$

$\omega_1 = 0.2\pi$ rad/s and

$\omega_2 = 0.5\pi$ rad/s

Thus

$$\Omega_1 = \frac{2}{T}\tan\frac{\omega_1}{2} = 0.65$$

$$\Omega_2 = \frac{2}{T}\tan\frac{\omega_2}{2} = 2$$

$$\epsilon = \sqrt{\left(\frac{1}{A_1}\right)^2 - 1} = 0.828$$

$$N \geq \frac{\cosh^{-1}\left(\frac{1}{\epsilon}\left(\left(\frac{1}{A_2}\right)^2 - 1\right)^{\frac{1}{2}}\right)}{\cosh^{-1}\left(\frac{\Omega_2}{\Omega_1}\right)} = 1.77$$

therefore $N \cong 2$

$\Omega_c = \Omega_1$

$$Y_N = Y_2 = \frac{1}{2}\left[\left\{\left(\frac{1}{\epsilon^2} + 1\right)^{1/2} + \frac{1}{\epsilon}\right\}^{1/N} - \left\{\left(\frac{1}{\epsilon^2} + 1\right)^{1/2} + \frac{1}{\epsilon}\right\}^{-1/N}\right] = 0.53$$

thus $c_1 = {Y_2}^2 + cos^2(\frac{\pi}{4}) = 0.78$

$b_1 = 2Y_2 \sin\left(\frac{\pi}{4}\right) = 0.75$

$B_1 = (A_1 c_1) = 0.6$

therefore

$$H_a(s) = \frac{0.2535}{s^2 + 0.4875s + 0.33}$$

and thus

$$H(z)=\frac{0.2535}{\left(2\frac{1-z^{-1}}{1+z^{-1}}\right)^2+0.4875\left(2\frac{1-z^{-1}}{1+z^{-1}}\right)+0.33}$$

$$H(z)=\frac{0.2535z^{-2}+0.507z^{-1}+0.2535}{3.355z^{-2}-7.34z^{-1}+5.305}$$

16. Determine H(z) using the impulse invariant technique for the analog system function

AKTU 2016-17

$$H(s)=\frac{1}{(s+0.5)(s^2+0.5s+2)}$$

Ans. After partial fraction

$$H(s)=\frac{1/2}{(s+0.5)}-\frac{s/2}{s^2+0.5s+2}$$

$$H1(s)=\frac{1/2}{(s+0.5)},H2(s)=\frac{s/2}{s^2+0.5s+2}$$

Thus $H(s)=H1(s)-H2(s)$

From $H1(s)H1(z)=\frac{1/2}{1-e^{j0.5T}z^{-1}}$

$$H2(s)=\frac{1}{2}\left[\left(\frac{s+\left(\frac{1}{4}\right)}{\left(s+\frac{1}{4}\right)^2+\left(\frac{\sqrt{31}}{4}\right)^2}\right)-\frac{1}{\sqrt{31}}\left(\frac{\left(\frac{\sqrt{31}}{4}\right)}{\left(s+\frac{1}{4}\right)^2+\left(\frac{\sqrt{31}}{4}\right)^2}\right)\right]$$

$H2(s)$can be converted to $H2(z)$ using following two relations

$$\frac{s+a}{(s+a)^2+b^2}\to\frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$$\frac{b}{(s+a)^2+b^2}\to\frac{e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

17. Convert the analog filter with system $H(s)=\frac{s+0.1}{(s+0.1)^2+9}$ functioninto digital filter with a $\omega_r=\frac{\pi}{4}$ resonant frequency of using bilinear transformation.

AKTU 2016-17

Ans.

For the given system $\Omega_c = 3$.

$$\Omega_c = \frac{2}{T}\tan\left(\frac{\omega_r}{2}\right)$$

$$T = \frac{2}{\Omega_c}\tan\left(\frac{\omega_r}{2}\right) = \frac{2}{3}\tan\left(\frac{\pi}{8}\right) = 0.3s$$

By using bilinear transformation

$$H(z) = H_a(s)\Big|_{s=\frac{2(1-z^{-1})}{T(1+z^{-1})}}$$

$$H(z) = \frac{\frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})} + 0.1}{\left(\frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})} + 0.1\right)^2 + 9}$$

Section-C- 15 marks questions

18. Explain frequency warping effect. How this problem is overcome in Bilinear transformation method of IIR filter design? Apply Bilinear transformation technique to transform the analog transfer function:

UPTU 2011-12, 2014-15, 2015-16, AKTU 2016-17, 2018-19

$$H_a(s) = \frac{1}{(s+1)(s^2+s+1)}.$$

Ans. The discrete-time filter behaves at frequency ω the same way that the continuous-time filter behaves at frequency (2/T) tan(ωT/2). Specifically, the gain and phase shift that the discrete-time filter has at frequency ω is the same gain and phase shift that the continuous-time filter has at frequency (2/T) tan(ωT/2). This means that every feature, every"bump" that is visible in the frequency response of the continuous-time filter is also visible in the discrete-time filter, but at a different frequency. For low frequencies (that is, when $\omega << 2/T$ or $\Omega << 2/T$), $\omega \approx \Omega$.

One can see that the entire continuous frequency range

$-\infty < \Omega < \infty$

is mapped onto the fundamental frequency interval

$-(\pi/T) < \Omega < (\pi/T)$

The continuous-time filter frequency Ω=0 corresponds to the discrete-time filterfrequency ω=0 and the continuous-time filter frequency Ω= ±∞ correspond to the discrete-time filter frequency ω= ±π/T.

One can also see that there is a nonlinear relationship between Ω and ω. This effect of the bilinear transform is called frequency warping.

Solution to Frequency Warping

The continuous-time filter can be designed to compensate for this frequency warping by setting $\Omega = (2/T)\tan(\omega T/2)$ for every frequency specification that the designer has control over (such as corner frequency or center frequency). This is called pre-warping the filter design.

When designing a digital filter as an approximation of a continuous time filter, the frequency response (both amplitude and phase) of the digital filter can be made to match the frequency response of the continuous filter at frequency ω_0 if the following transform is substituted into the continuous filter transfer function.

The main advantage of the warping phenomenon is the absence of aliasing distortion of the frequency response characteristic, such as observed with Impulse invariance. It is necessary, however, to compensate for the frequency warping by pre-warping the given frequency specifications of the continuous-time system. These pre-warped specifications may then be used in the bilinear transform to obtain the desired discrete-time system.

Given

$$H_a(s) = \frac{1}{(s+1)(s^2+s+1)}$$

By using Bilinear transformation to obtain H(z) from $H_a(s)$, 's' is to be replaced by

$$s \leftarrow \frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})}$$

Therefore

$$H(z) = \frac{T^3(1+z^{-1})^3}{\left((2+T)-(2-T)z^{-1}\right)\left((4+2T+2T^2)-(8-2zT^2)z^{-1}+(4-2T+T^2)z^{-2}\right)}$$

19. Design digital Butterworth filter from the specification given below:

$$0.8 \le \left|H\left(e^{j\omega}\right)\right| \le 1 \qquad 0 \le |\omega| \le 0.2\pi$$
$$\left|H\left(e^{j\omega}\right)\right| \le 0.2 \qquad 0.6\pi \le |\omega| \le \pi$$

AKTU 2018-19, 2020-21, 2022-23

Ans. Comparing the given constraint with the standard form of Butterworth filter

$$\omega_1 = 0.2\pi \qquad \omega_2 = 0.6\pi$$

$$A_1 = 0.8 \qquad A_2 = 0.2$$

In order to obtain $H_a(j\Omega)$, we must determine the values of the parameters N and Ω_c.

The analog frequency ratio

$$\frac{\Omega_2}{\Omega_1} = \frac{\tan(\omega_2/2)}{\tan(\omega_1/2)} = \frac{1.376}{0.3249} = 4.235$$

Thus

$$N \geq \frac{1}{2}\frac{\log\left[\left(\frac{1}{A_2}\right)^2 - 1\right] / \left[\left(\frac{1}{A_1}\right)^2 - 1\right]}{\log\left(\frac{\Omega_2}{\Omega_1}\right)}$$

$$\geq \frac{1}{2}\frac{\log[1/0.04 - 1] / [1/0.64 - 1]}{\log(4.235)}$$

$$\geq 1.3$$

Therefore the required filter order N = 2.

The analog cutoff frequency

$$\Omega_c = \frac{2}{T}\frac{\tan(\omega_1/2)}{\left[\left(\frac{1}{A_1}\right)^2 - 1\right]^{1/2N}}$$

$$= \frac{2}{T}\frac{0.3249}{\left[\frac{1}{0.64} - 1\right]^{\frac{1}{4}}}$$

$$= \frac{2}{T}(0.3752)$$

Taking $B_1 = 1$

$$H_a(s) = \frac{\Omega_c^{\,2}}{s^2 + b_1\Omega_c s + \Omega_c^{\,2}}$$

Where $b_1 = 2\sin\left(\frac{\pi}{4}\right) = 1.4142$.

On applying bilinear transformation

$$H(z) = \frac{A_{10}(1 + z^{-1})^2}{1 + a_{11}z^{-1} + a_{12}z^{-2}}$$

$$A_{10} = \left(\frac{\Omega_c T}{2}\right)^2 [\left(\frac{\Omega_c T}{2}\right)^2 c_1 + \left(\frac{\Omega_c T}{2}\right) b_1 + 1]^{-1}$$

$$= 0.0842.$$

$$a_{11} = 2[\left(\frac{\Omega_c T}{2}\right)^2 c_1 - 1][\left(\frac{\Omega_c T}{2}\right)^2 c_1 + \left(\frac{\Omega_c T}{2}\right) b_1 + 1]^{-1}$$

$$= -1.0281.$$

$$a_{12} = [\left(\frac{\Omega_c T}{2}\right)^2 c_1 - \left(\frac{\Omega_c T}{2}\right) b_1 + 1][\left(\frac{\Omega_c T}{2}\right)^2 c_1 + \left(\frac{\Omega_c T}{2}\right) b_1 + 1]^{-1}$$

$$= 0.3651.$$

therefore

$$H(z) = \frac{0.0842(1 + z^{-1})^2}{1 - 1.0281z^{-1} + 0.3651z^{-2}}$$

20. Design digital Butterworth filter to meet the constraints :

$$0.9 \le |H(e^{j\omega})| \le 1 \qquad 0 \le |\omega| \le 0.25\pi$$

$$|H(e^{j\omega})| \le 0.2 \qquad 0.6\pi \le |\omega| \le \pi$$

using(i) Bilinear Transformation Technique and

(ii) Impulse Invariance Transformation Technique.**UPTU 2013-14, AKTU 2017-18**

Ans.Same as previous Question.

21. Derive the mathematical expression for impulse invariance technique. Discuss its advantages and how it can be taken care of. **UPTU 2011-12, AKTU 2022-23**

Ans. The impulse-invariant method converts analog filter transfer functions to digital filter transfer functions in such a way that the impulse response is the same (invariant) at the sampling instants. Thus, if γ(t) denotes the impulse-response of an analog (continuous-time) filter, then the digital (discrete-time) filter given by the impulse-invariant method will have impulse response γ(nT), where T denotes the sampling interval in seconds. Moreover, the order of the filter is preserved, and IIR analog filters map to IIR digital filters. However, the digital filter's frequency response is an aliased version of the analog filter's frequency response.

To derive the impulse-invariant method, we begin with the analog transfer function

$$\Gamma_a(s) \triangleq \frac{B_a(s)}{A_a(s)} \triangleq \frac{b_a(0)s^{N-1} + b_a(1)s^{N-2} + \cdots + b_a(N-2)s + b_a(N-1)}{s^N + a_a(1)s^{N-1} + \cdots + a_a(N-1)s + a_a(N)}$$

and perform a partial fraction expansion (PFE) down to first-order terms

$$\Gamma_a(s) = \sum_{i=1}^{N} \frac{K_i}{s - s_i},$$

Where s_i is the i th pole of the analog system, and K_i is its residue. Assume that the system is at least marginally stable so that there are no poles in the right-half plane (re{s_i} ≤ 0). Such a PFE is always possible when Γ(s) is a strictly proper transfer function (more poles than zeros). Performing the inverse Laplace transform on the partial fraction expansion we obtain the impulse response in terms of the system poles and residues:

$$\gamma_a(t) = \sum_{i=1}^{N} K_i e^{s_i t}, \quad t \geq 0.$$

We now sample at intervals of T seconds to obtain the digital impulse response

$$\gamma_d(n) \stackrel{\Delta}{=} \gamma_a(nT) = \sum_{i=1}^{N} K_i e^{s_i n T}, \quad n = 0, 1, 2, \ldots.$$

Taking the z transform gives the digital filter transfer function designed by the impulse-invariant method:

$$\Gamma_d(z) = \sum_{i=1}^{N} \frac{K_i}{1 - e^{s_i T} z^{-1}} \stackrel{\Delta}{=} \frac{B_d(z)}{A_d(z)}$$

We see that the s -plane poles s_i have mapped to the z -plane poles

$$\boxed{z_i \stackrel{\Delta}{=} e^{s_i T}}$$

and the residues have remained unchanged. Clearly we must have $-\pi < \text{im}\{s_i\}T < \pi$, i.e., the analog poles must lie within the bandwidth spanned by the digital sampling rate $f_s = 1/T$. Otherwise, the pole angle im{s_i}T will be aliased into the interval [-π, π). Stability is preserved since

re{s_i} ≤ 0 ⇔ $|z_i| \leq 1$.

Advantages:

- Stable design
- Analog frequency and digital frequency are linearly related

Disadvantage

- Aliasing
- Useful only when the analog filter is band-limited (LPF and BPF)

22. Compare the characteristics of Butterworth and Chebyshev filter. Determine the parameters of a Chebyshev filter for which $A_1 = 1/2^{1/2}$, $A_2 = 0.1$, $\Omega_1 = 2$ rad/s and $\Omega_2 = 4$ rad/s.

UPTU 2014-15, AKTU 2019-20

Ans. In digital signal processing, we come across digital filters which are to be designed using analog filters. From these analog filters, Butterworth and Chebyshev filters are the most popular one.

Some of the important differences are as follows:

Magnitude response vs frequency curve: The magnitude response |H(jw)| of the butterworth filter decreases with increase in frequency from 0 to infinity, while the magnitude response of the Chebyshev filter fluctuates or show ripples in the passband and stopband depending on the type of the filter.

Width of Transition band: The width of the transition band is more in Butterworth filter compared to the Chebyshev filter.

Location of the poles: The poles of a Butterworth filter lies only on a circle while that of the Chebyshev filter lies on an ellipse, which can be easily concluded on looking at the poles formula for both types of filters.

No. Of Components required for implementing the filter: The number of poles in Butterworth filter is more compared to that of the Chebyshev filter of same specifications, this means that the order of Butterworth filter is more than that of a Chebyshev filter. This fact can be used for practical implementation, since the number of components required to construct a filter of same specification can be reduced significantly.

Given

$A_1 = 1/2^{1/2}$

$A_2 = 0.1$

$\Omega_1 = 2$ rad/s and

$\Omega_2 = 4$ rad/s

Thus

$$\epsilon = \frac{1}{\sqrt{\left(\frac{1}{A_1}\right)^2 - 1}} = 1$$

$$N \geq \frac{\cosh^{-1}\left(\frac{1}{\epsilon}\left(\left(\frac{1}{A_2}\right)^2 - 1\right)^{\frac{1}{2}}\right)}{\cosh^{-1}\left(\frac{\Omega_2}{\Omega_1}\right)} = 2.27$$

therefore $N \cong 3$

$\Omega_c = \Omega_1$

$$Y_N = Y_3 = \frac{1}{2}\left[\left\{\left(\frac{1}{\epsilon^2} + 1\right)^{1/2} + \frac{1}{\epsilon}\right\}^{1/N} - \left\{\left(\frac{1}{\epsilon^2} + 1\right)^{1/2} + \frac{1}{\epsilon}\right\}^{-1/N}\right] = 0.298$$

thus $c_0 = Y_3 = 0.298$

$c_1 = {Y_3}^2 + cos^2(\frac{\pi}{6}) = 0.84$

$b_1 = 2Y_3 \sin\left(\frac{\pi}{6}\right) = 0.298$

$B_1 = B_0 = (c_0\ c_1)^{1/2} = 0.5.$

23. Determine H(z) for a Butterworth filter satisfying the following constraints

AKTU 2016-17

$$\sqrt{0.5} \leq \left|H(e^{j\omega})\right| \leq 1 \qquad 0 \leq \omega \leq \frac{\pi}{2}$$

$$\left|H(e^{j\omega})\right| \leq 0.2 \qquad \frac{3\pi}{4} \leq \omega \leq \pi$$

Ans. Comparing the given constraint with the standard form of Butterworth filter

$$\omega_1 = 0.5\pi \qquad \omega_2 = 0.75\pi$$

$$A_1 = 0.707 \qquad A_2 = 0.2$$

In order to obtain $H_a(j\Omega)$, we must determine the values of the parameters N and Ω_c.

Same as Section-C-Q2

Additional Questions

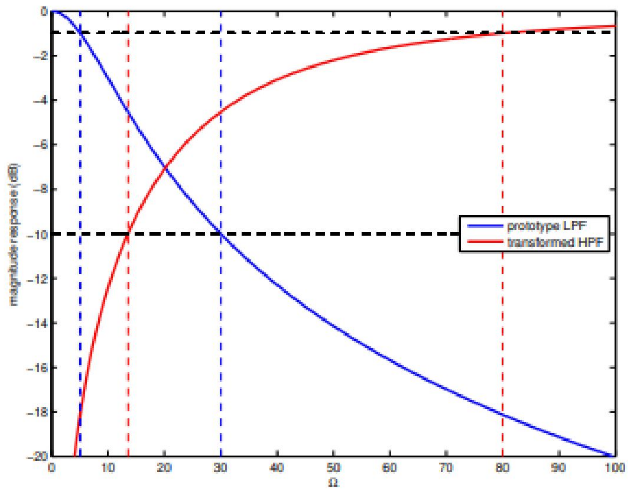
13. Derive the frequency-transformation for converting a prototype digital low pass filter into a digital high pass filter. **UPTU 2012-13**

Ans. For lowpass to highpass transformations, we use $s \rightarrow \frac{\Omega_p \hat{\Omega}_p}{s}$

Substituting s = jΩ on the lefthand side and s = j$\hat{\Omega}$ on the righthand side, we can relate the frequencies of the prototype and transformed systems as $\Omega = \frac{\Omega_p \hat{\Omega}_p}{\hat{\Omega}}$.

Given Ωp and Ωs of the prototype filter, we can pick our desired value of $\hat{\Omega}_p$ for our transformed highpass filter and then compute the stop band frequency edge $\hat{\Omega}_s = \frac{\Omega_p \hat{\Omega}_p}{\Omega_s}$.

We assume a symmetric magnitude response, so the minus signs can be ignored.



14. What is the equation for order of Butterworth filter? **AKTU 2017-18**

Ans.

$$N \geq \frac{1}{2} \frac{\log\left(\frac{\left\{\left[\frac{1}{A_2}\right]^2 - 1\right\}}{\left\{\left[\frac{1}{A_1}\right]^2 - 1\right\}}\right)}{log\left(\frac{\Omega_2}{\Omega_1}\right)}$$

15. Use bilinear transformation to convert low pass filter, H(s) = 1/ $s^2 + \sqrt{2}s + 1$ into a high pass filter with pass band edge at 100 Hz and F_S = 1 kHz. **AKTU 2017-18, 2020-21**

Ans. Transform low pass filter to high pass filter by replacing s to Ω_c/s

Then for High pass filter

$$H(s) = \frac{1}{\left(\frac{\Omega_c}{s}\right)^2} + \sqrt{2}\frac{\Omega_c}{s} + 1 = \frac{s^2}{\Omega_c{}^2} + \sqrt{2}\frac{\Omega_c}{s} + 1$$

$$\Omega_c = 2\pi * \left(\frac{100}{1000}\right) = 0.2\pi\, rad/sec$$

$$\mathrm{T} = \frac{1}{\mathrm{Fs}} = .001 \text{ sec}$$

$$\text{thus } H(s) = \frac{s^2}{(0.2\pi)^2} + \sqrt{2}\frac{0.2\pi}{s} + 1$$

by using Bilinear transform replace 's' with $2000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$

$$\text{thus} H(z) = \frac{(2000\left(\frac{1-z^{-1}}{1+z^{-1}}\right))^2}{(0.2\pi)^2} + \sqrt{2}\frac{0.2\pi}{2000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} + 1$$

16. The system function of the analog filter is given as : **AKTU 2017-18, 2022-23**

$$H(s) = \frac{s + 0.1}{(s + 0.1)^2 + 16}$$

Obtain the system function of digital filter using bilinear transformation which is resonant at $\omega_r = \pi/2$

Ans. Same as Q10

17. Find the order of an active low pass Butterworth filter whose specifications are given as: A_{max} = 0.5dB at a pass band frequency (Ωp) of 200 radian/sec (31.8Hz), and A_{min} = 20dBat a stop band frequency (Ωs) of 800 radian/sec. Also design a suitable Butterworth filter to match these requirements. Assume sampling frequency is equal to 1000Hz .

Ans.

Firstly, the maximum pass band gain Amax = 0.5dB which is equal to a gain of 0.94(0.5dB = 20log (1/A1)) at a frequency (Ωp) of 200 rads/s.
Secondly, the minimum stop band gain Amin = 20dB which is equal to a gain of 01 (20dB = 20log (1/A2)) at a stop band frequency (Ωs) of 800 rads/s
So Ωs/Ωp = 4.
Thus $N = \frac{1}{2}\frac{\log(810.7)}{\log(4)} = 2.42 \cong 3$
So the filter order is 3.

$$\Omega_c = \frac{\Omega_p}{\left(\left(\frac{1}{A1^2}\right) - 1\right)^{\frac{1}{2N}}} = 0.284$$

Taking B0=B1 = 1 and c0=c1=1

$$H_a(s) = \frac{\Omega_c^{\,2}}{s^2 + b_1\Omega_c s + \Omega_c^{\,2}}$$

Where $b_1 = 2\sin\left(\frac{\pi}{6}\right) = 1.732.$

Then $H_a(s) = \frac{0.08}{s^2+0.49s+0.08}$

18. Find the order and cut off frequency of a digital filter with the following specification

$$0.89 \le \left|H\left(e^{j\omega}\right)\right| \le 1, \qquad 0 \le \omega \le 0.4\pi$$

$$\left|H\left(e^{j\omega}\right)\right| \le 0.18, \qquad 0.6\pi \le \omega \le \pi$$

use the impulse invariance method? **AKTU 2018-19**

19. The system function of analog filter is given by

$$H(s) = \frac{s+0.1}{(s+0.1)^2+16}$$

Obtain the system function of digital filter by using impulse invariant technique. Assume T=1sec. **AKTU 2019-20**

20. Design a Butterworth low pass analog filter for the following specification:

i) Pass band gain required:0.9
ii) Frequency up to which pass band gain must remain more or less steady :100 rad/sec
iii) Gain in attenuation band:0.4
iv) Frequency from which the attenuation must start: 200 rad/sec **AKTU 2019-20**

21. Compute the poles of an analog Chebyshev filter transfer function that satisfies the constraints:

$$0.8 \le \left|H\left(e^{j\omega}\right)\right| \le 1, \qquad 0 \le \omega \le 0.2\pi$$

$$\left|H\left(e^{j\omega}\right)\right| \le 0.2, \qquad 0.32\pi \le \omega \le \pi$$

AKTU 2020-21

And determine H(s) and hence obtain H(z) using Bilinear transformation. Assume T=1 sec.

22. Explain the phenomenon of digital frequency transformation. **AKTU 2020-21**

23. What are the differences between impulse invariant transformation and bilinear transformation method? **AKTU 2020-21**

24. Evaluate the value of C3(x), Chebyshev Polynomial. **AKTU 2021-22**

25. Design Digital Butterworth filter to satisfy the following constraints using bilinear transformation method, the sampling Interval is 2 second: assume missing data if required: **AKTU 2021-22**

$0.52 \leq |H(e^{jw})| \leq 1, \quad 0 \leq w \leq \pi/2$

$|H(e^{jw})| \leq 0.1, \quad 3\pi/4 \leq w \leq \pi$

26. Design Chebyshev Digital LPF filter to satisfy the following constraints using Impulse Invariant method. **AKTU 2021-22**

$0.9 \leq |H(e^{jw})| \leq 1, 0 \leq w \leq 0.25\pi$

$|H(e^{jw})| \leq 0.24, 0.5\pi \leq w \leq \pi$

27. Design Chebyshev Digital LPF filter to satisfy the following constraints using Bilinear Transformation method, assume that the sampling time is one second. **AKTU 2021-22**

$0.707 \leq |H(e^{jw})| \leq 1, 0 \leq w \leq 0.2\pi$

$|H(e^{jw})| \leq 0.1, 0.5\pi \leq w \leq \pi$

UNIT-3

(Finite Impulse Response Filter (FIR) Design)

Section A: 2 marks questions

Q1. What is the principal of designing FIR filter using windows? **UPTU 2010-11**

Ans. Windowing is the quickest method for designing an FIR filter. A windowing function simply truncates the ideal impulse response to obtain a causal FIR approximation that is non causal and infinitely long. Smoother window functions provide higher out-of band rejection in the filter response. However this smoothness comes at the cost of wider stopband transitions.

Various windowing method attempts to minimize the width of the main lobe (peak) of the frequency response. In addition, it attempts to minimize the side lobes (ripple) of the frequency response.

Q2. Determine the frequency response of symmetric Hann window given by:

$$w(n) = \begin{cases} \frac{1}{2}(1+\cos\frac{n\pi}{M}) & -m \le n \le M \\ 0 & \text{Otherwise} \end{cases}$$

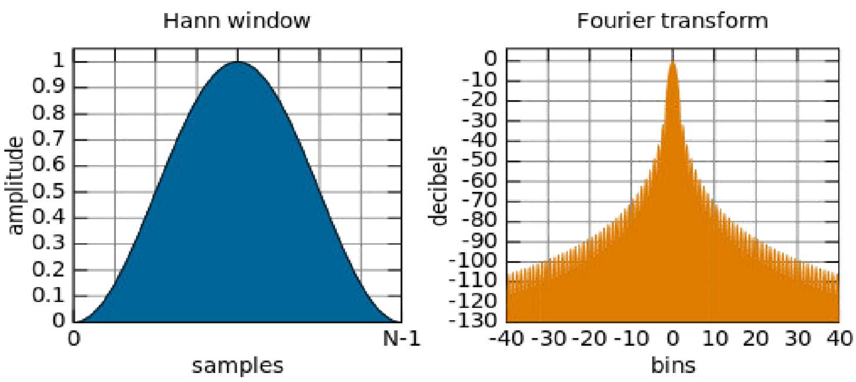
UPTU 2011-12

Ans.

The Hann window named after Julius von Hann and also known as the **Hanning**(for being similar in name and form to the Hamming window), **von Hann** and the**raised cosine window** is defined by (with hav for the haversine function):

$$w(n) = 0.5\left(1-\cos\left(\frac{2\pi n}{N-1}\right)\right) = hav\left(\frac{2\pi n}{N-1}\right)$$

The ends of the cosine just touch zero, so the side-lobes roll off at about 18 dB per octave

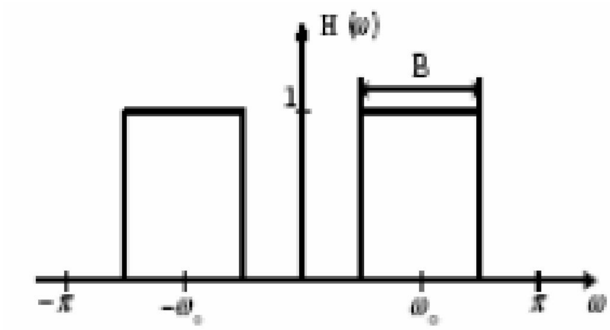


Q3. The frequency response of an ideal bandpass filter is given by

$$H(\omega) = \begin{cases} 0 & |\omega| \le \frac{\pi}{8} \\ 1 & \frac{\pi}{8} < |\omega| < \frac{3\pi}{8} \\ 0 & \frac{3\pi}{8} < |\omega| < \pi \end{cases}$$

Determine its impulse response. **UPTU 2012-13**

Ans. Impulse response is the inverse fourier of the given frequency response.



$$\omega_c = \pi/4$$

$$F^{-1}(H(\omega)) = h(n) = 2.\cos\left(\frac{\pi}{4}.n\right).\left(\frac{1}{4}sinc\left(\frac{n}{4}\right)\right)\Bigg|_{B=\pi/2}$$

Section B: 10 marks questions

Q4. What is the reason that FIR filters are always stable? Also write the properties of FIR filter. **UPTU 2010-11**

Ans.

Difference equation of FIR filter of length M is given as

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k) \qquad \ldots\ldots\ldots.1$$

And the coefficient b_k are related to unit sample response as

$$H(n) = \begin{cases} b_n & for \quad 0 \le n \le M-1 \\ 0 & \qquad otherwise \end{cases}$$

We can expand this equation as

$$y(n) = b_0x(n) + b_1x(n-1) + \cdots \ldots\ldots\ldots\ldots\ldots. + b_{M-1}x(n-(M-1)) \qquad \ldots\ldots\ldots.2$$

System is stable only if system produces bounded output for every bounded input. This is stability definition for any system.

Here h(n)={b0, b1, b2, } of the FIR filter are stable. Thus y(n) is bounded if input x(n) is bounded. This means FIR system produces bounded output for every bounded input. Hence FIR systems are always stable.

The following are useful properties of FIR filters:

- They are always stable — the system function contains no poles. This is particularly useful for adaptive filters.
- They can have an exactly linear phase response. The result is no frequency dispersion, which is good for pulse and data transmission.
- Finite length register effects are simpler to analyse and of less consequence than for IIR filters.
- They are very simple to implement, and all DSP processors have architectures that are suited to FIR filtering.
- For large N (many filter taps), the FFT can be used to improve performance

Q5. Compare FIR and IIR filter. Show that for a linear phase FIR filter, the impulse response is given by: **UPTU 2011-12, 2014-15, 2015-16**

h(n)=±h(N-1-n), and hence classify the FIR filters.

Ans. IIR filter structures can therefore be far more computationally efficient than FIR filters, particularly for long impulse responses.

FIR filters are stable for h(n)• bounded, and can be made to have a linear phase response. IIR filters, on the other hand, are stable if the poles are inside the unit circle, and have a phase response that is difficult to specify. The general approach taken is to specify the magnitude response, and regard the phase as acceptable. This is a disadvantage of IIR filters.

A linear-phase filter is typically used when a causal filter is needed to modify a signal's magnitude-spectrum while preserving the signal's time-domain waveform as much as possible. Linear-phase filters have a symmetric impulse response, e.g.,

$$h(n) = h(N-1-n), \quad n = 0, 1, 2, \dots., N-1$$

The symmetric-impulse-response constraint means that linear-phase filters must be FIR filters, because a causal recursive filter cannot have a symmetric impulse response.

We will show that every real symmetric impulse response corresponds to a real frequency response times a linear phase term $e^{-j\alpha\omega T}$, where $\alpha = (N-1)/2$ is the slope of the linear phase. Linear phase is often ideal because a filter phase of the form $\Theta(\omega) = -\alpha\omega T$ corresponds to phase delay

$$P(\omega) \triangleq -\frac{\Theta(\omega)}{\omega} = -\frac{-\alpha\omega T}{\omega} = \alpha T = -\frac{(N-1)T}{2}$$

and group delay

$$D(\omega) \triangleq -\frac{\partial}{\partial\omega}\Theta(\omega) = -\frac{\partial}{\partial\omega}(-\alpha\omega T) = \alpha T = \frac{(N-1)T}{2}$$

That is, both the phase and group delay of a linear-phase filter are equal to $(N-1)/2$ samples of plain delay at every frequency. Since a length N FIR filter implements $(N-1)$ samples of delay, the value $(N-1)/2$ is exactly half the total filter delay. Delaying all frequency components by the same amount preserves the waveshape as much as possible for a given amplitude response.

Q6. Use the rectangular window to design a linear phase FIR filter of order N=24 to approximate the following frequency response magnitude-

$$\left|H_d(e^{j\omega})\right| = \begin{cases} 1, & |\omega| \leq 0.2\pi \\ 0, & 0.2\pi \leq |\omega| \leq \pi \end{cases}$$

Ans. Let's consider the order of the filter N an odd number thus N≈25 For the given magnitude response

$$h_d(n) = \frac{\sin 0.2\pi n}{\pi n}; \qquad n \neq 0$$

and rectangular window

$$w(n) = \begin{cases} 1, & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}$$

the linear phase FIR filter coefficients h(n) for N=25 are computed as

n=	$h_d(n)$=	w(n)=	h(n)= $h_d(n)$x w(n)
0	0.2	1	0.2
1	0.187111	1	0.187111
2	0.151411	1	0.151411
3	0.100993	1	0.100993
4	0.04688	1	0.04688
5	0.000101	1	0.000101
6	-0.03112	1	-0.03112
7	-0.04324	1	-0.04324
8	-0.03789	1	-0.03789
9	-0.02088	1	-0.02088
10	-0.0001	1	-0.0001

11	0.016935	1	0.016935
12	0.025209	1	0.025209

Therefore the complete 25 points of h(n) are

n=0	0.025209
1	0.016935
2	-0.0001
3	-0.02088
4	-0.03789
5	-0.04324
6	-0.03112
7	0.000101
8	0.04688
9	0.100993
10	0.151411
11	0.187111
12	0.2
13	0.187111
14	0.151411
15	0.100993
16	0.04688
17	0.000101
18	-0.03112
19	-0.04324
20	-0.03789
21	-0.02088
22	-0.0001
23	0.016935
24	0.025209

Q7. Design an FIR filter to meet the following specifications: **AKTU 2017-18**

Passband edge = 2 kHz

Stop band edge = 5 kHz

Stop band Attenuation = 42 dB

Sampling Frequency = Fs = 20 kHz

Use Hanning window.

Ans.

Given F_s = 20kHz

f_1 = 2kHz ➔ $\omega_1 = 4x10^3\pi / F_s = 0.2\pi$ [normalized]

f_2 = 5kHz ➔ $\omega_2 = 10^4\pi / F_s = 0.5\pi$

Δf=3/20=0.15

$\omega_c = (\omega_1 + \omega_2)/2 = 0.35\pi$

A_2 = 42 dB

$$N = \frac{A - 7.95}{28.72\Delta f}$$

N= 7.95 ≈ 9 (taking N odd)

Since from given specifications the filter is low pass filter thus

$$h_d(n) = \frac{\sin 0.35\pi n}{\pi n}; \qquad n \neq 0$$

Hanning window function

$$w(n) = \begin{cases} \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{N-1}\right)\right], & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

compute h(n) same as q6.

Q8. Discuss the FIR filter design technique using windows, with mathematical expressions. Discuss the disadvantages of this method.

Ans. an FIR filter is one that is described by the difference equation

$$y[n] = \sum_{l=0}^{M} b_l x[n-l]$$

and by the transfer function

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \sum_{l=0}^{M} b_l e^{-j\omega l}$$

We address the problem of designing an FIR filter that meets specifications of limited deviation
from the ideal response in specified frequency bands.
The window design method does not produce filters that are optimal (in the sense of meeting the design specifications in the most computationally-efficient fashion), but the method is easy to understand and does produce filters that are

reasonably good. Of all the hand-design methods, the window method is the most popular and effective.

In brief, in the window method we develop a causal linear-phase FIR filter by multipying an ideal filter that has an infinite-duration impulse response (IIR) by a finite-duration window function:

$$h[n] = h_d[n]w[n]$$

Where h(n) is the practical FIR filter, $h_d(n)$ is the ideal IIR prototype filter, and w(n) is the finite-duration window function. An important consequence of this operation is that the DTFTs of $h_d(n)$ and w(n) undergo circular convolution in frequency:

$$H(e^{j\omega}) = \frac{1}{2\pi}\oint_{2\pi} H_d(e^{j\theta})W(e^{j(\omega-\theta)})d\theta$$

Q9. Design a low pass digital FIR filter having following specifications:

$$0.99 \le |H(e^{j\omega})| \le 1.01 \qquad 0 \le |\omega| \le 0.19\pi$$

$$|H(e^{j\omega})| \le 0.01 \qquad 0.21\pi \le |\omega| \le \pi$$

Use Hanning window. Assume $\omega_c = 0.2\pi$, express the impulse response $h_d(n)$.

Ans. Same as Q7

Q10. The desired frequency response of a low pass filter is given by

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega}, & |\omega| \le \frac{\pi}{4} \\ 0, & Otherwise \end{cases}$$

Design the filter using hamming window. **UPTU 2012-13**

Ans. For the given system

$$h_d(n) = \frac{\omega_c}{\pi}.sinc\left(\frac{\omega_c n}{\pi}\right)$$

$$\omega_c = \frac{\pi}{4}$$

$$then\ h_d(n) = \frac{1}{4}.sinc\left(\frac{n}{4}\right)$$

Hamming window function

$$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right), & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

compute h(n) same as q3.

Q11. The desired response of a low pass filter is

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega}; & \frac{-3\pi}{4} \le \omega \le \frac{3\pi}{4} \\ 0 \quad ; & \frac{3\pi}{4} < |\omega| \le \pi \end{cases}$$

Determine $H(e^{j\omega})$ for M = 7 using Hamming Window. **UPTU 2013-14**

Ans. Impulse response for the given system

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n} d\omega$$

$$= \begin{cases} \frac{\sin(n-\tau)}{\pi(n-\tau)} & for \quad n \ne \tau \\ \frac{1}{\pi} & for \quad n = \tau \end{cases}$$

Hamming window function

$$\mathrm{w(n)} = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right), & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

compute h(n) same as q3.

Q12. Design a low-pass filter with the following desired frequency response

AKTU 2016-17

$$H_d(e^{jw}) = \begin{cases} e^{-j2\omega}, & \frac{-\pi}{4} \le \omega \le \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |\omega| < \pi \end{cases}$$

and using window function

$$w(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & otherwise \end{cases}$$

Ans. For N=5 Same as Q7

Section C: 15 marks questions

Q13. Explain Gibb's Phenomenon. Find the response of rectangular window and explain it.

UPTU 2010-11, 2011-12, 2014-15, AKTU 2016-17, 2017-18

Ans. According to our theory, with a complete set of basis functions we can represent any function exactly. We furthermore know how to obtain the best approximation if we use only a finite set of functions. Interestingly, even the best approximation can still have some substantial errors. Consider the error in the square-wave series. Observe that there is a jump just before the point of discontinuity. As it turns out, **no matter how large n** is, this error remains, and it has an amplitude of about 9% of the discontinuity. As **n** gets larger and larger, this wiggle becomes narrower and moves closer to the point of the discontinuity, but it **never goes away.** This overshoot phenomenon is known as the **Gibbs phenomenon.**

The (zero-centered) *rectangular window* may be defined by

$$w_R(n) \triangleq \begin{cases} 1, & -\frac{M-1}{2} \leq n \leq \frac{M-1}{2} \\ 0, & otherwise \end{cases}$$

where M is the window length in samples (assumed odd for now). A plot of the rectangular window appears in Fig for length M = 21. It is sometimes convenient to define windows so that their dc gain is 1, in which case we would multiply the definition above by $1/M$.

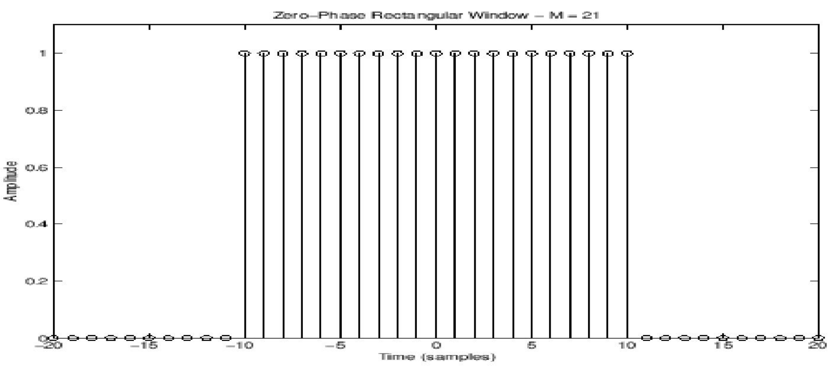


Figure: The rectangular window.

To see what happens in the frequency domain, we need to look at the DTFT of the window:

$$W_R(\omega) = DTFT_\omega(w_R) \triangleq \sum_{n=-\infty}^{\infty} w_R(n)e^{-j\omega n}, \qquad \omega \in [-\pi, \pi)$$

$$= \sum_{n=-\frac{M-1}{2}}^{\frac{M-1}{2}} e^{-j\omega n} = \frac{e^{-j\omega\frac{M-1}{2}} - e^{-j\omega\frac{M+1}{2}}}{1 - e^{-j\omega}}$$

where the last line was derived using the closed form of a geometric series:

$$\sum_{n=L}^{U} r^n = \frac{r^L - r^{U+1}}{1 - r}$$

We can factor out linear phase terms from the numerator and denominator of the above expression to get

$$W_R(\omega) = \frac{e^{-j\omega\frac{1}{2}}}{e^{-j\omega\frac{1}{2}}}\left[\frac{e^{j\omega\frac{M}{2}} - e^{-j\omega\frac{M}{2}}}{e^{j\omega\frac{1}{2}} - e^{-j\omega\frac{1}{2}}}\right]$$

$$= \frac{\sin\left(M\frac{\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)} \triangleq M.asinc_M(\omega)$$

where $asinc_M(\omega)$ denotes the *aliased sinc function*

$$asinc_M(\omega) \triangleq \frac{\sin\left(M\frac{\omega}{2}\right)}{M.\sin\left(\frac{\omega}{2}\right)}$$

(also called the *Dirichlet function* or *periodic sinc function*). This (real) result is for the*zero-centered* rectangular window. For the causal case, a linear phase term appears:

$$W_R^c(\omega) = e^{-j\omega\frac{M-1}{2}}.M.asinc_M(\omega)$$

The term "aliased sinc function'' refers to the fact that it may be simply obtained by *sampling*the length-τ *continuous-time* rectangular window, which has Fourier transform sinc $(f\tau) \triangleq \sin(\pi f\tau)/(\pi f\tau)$ (given amplitude $1/\tau$ in the time domain). Sampling at intervals of Tseconds in the time domain corresponds to aliasing in the frequency domain over the interval $[0, 1/T]$ Hz, and by direct derivation, we have found the result. It is interesting to consider what happens as the window duration increases continuously in the time domain: themagnitude spectrum can only change in discrete jumps as new samples are included, even though it is continuously parametrized in τ.

As the sampling rate goes to infinity, the aliased sinc function therefore approaches the *sinc function*

$$sinc(x) \triangleq \frac{\sin(\pi x)}{\pi x}$$

Specifically,

$$\lim_{\substack{\tau \to 0 \\ MT=\tau}} asinc_M(\omega T) = sinc(\tau f).$$

where$\omega = 2\pi f$

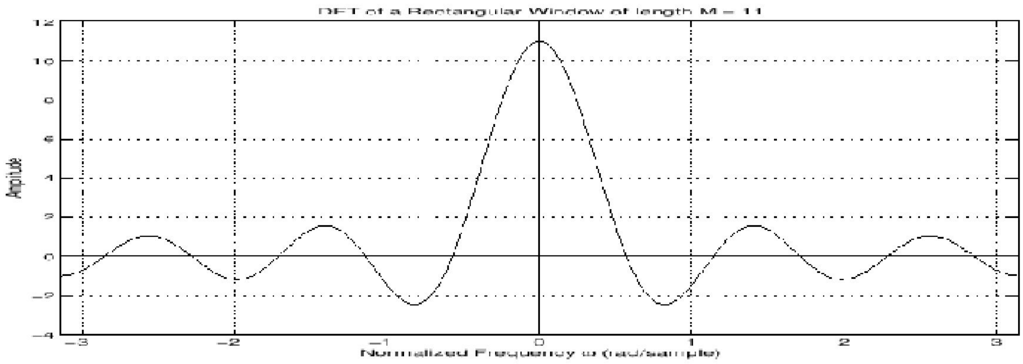


Figure : Fourier transform of the rectangular window.

Figure illustrates$W_R(\omega) = M.asinc_M(\omega)$ for M=11. Note that this is the complete window transform, not just its real part. We obtain real window transforms like this only for zero-centered, symmetric windows. Note that the phase of rectangular-window transform$W_R(\omega)$ is *zero* for$|\omega| < 2\pi/M$, which is the width of the *main lobe*. This is why zero-centered windows are often called *zero-phase windows*; while the phase actually alternates between 0 andπ radians, the πvalues occur only within side-lobes which are routinely neglected (in fact, the window is normally designed to ensure that all side-lobes can be neglected).

More generally, we may plot both the *magnitude* and *phase* of the window versus frequency, as shown in Figures and below. In audio work, we more typically plot the window transform magnitude on a *decibel (dB) scale*, as shown in Fig. below. It is common to normalize the peak of the dB magnitude to 0 dB, as we have done here.

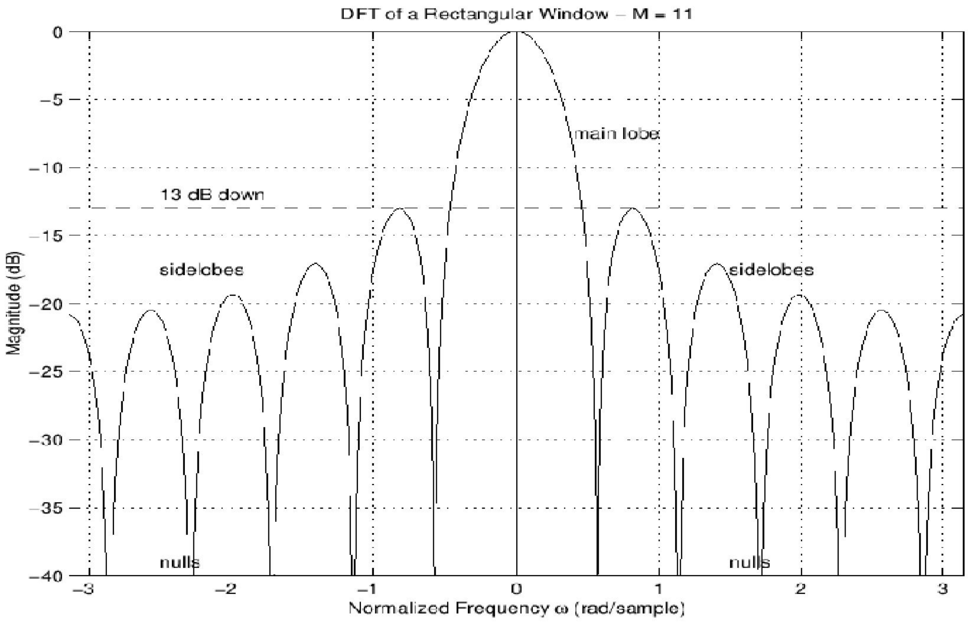


Figure : Magnitude (dB) of the rectangular-window transform.

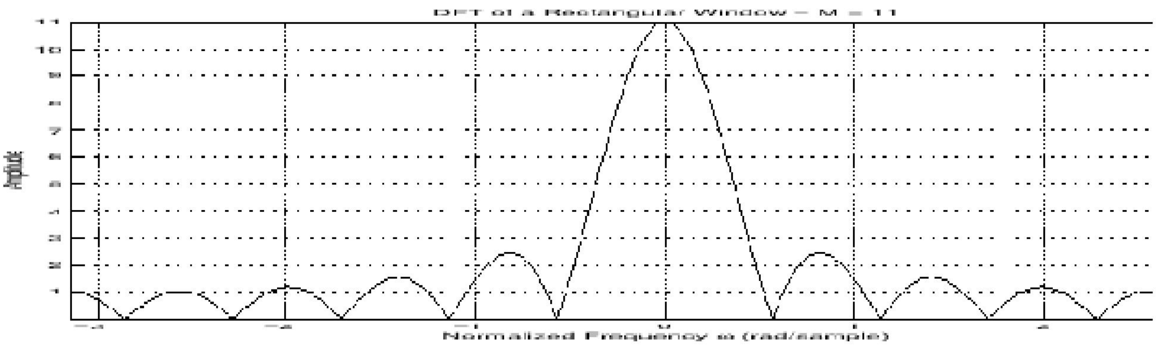


Figure : Magnitude of the rectangular-window Fourier transform.

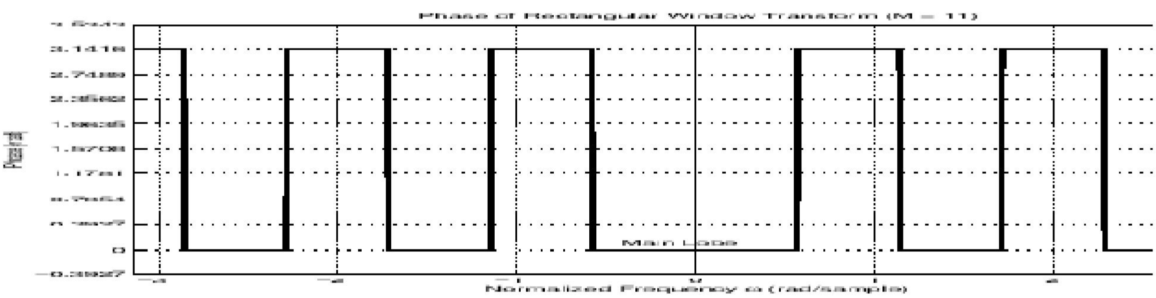


Figure : Phase of the rectangular-window Fourier transform.

RECTANGULAR WINDOW SIDE-LOBES

From Fig., we see that the main-lobe width is$2.2\pi/M = 4\pi/11 \approx 1.1$radian, and the side-lobe level is 13 dB down.

Since the DTFT of the rectangular window approximates the sinc function, which has an amplitude envelope proportional to $1/\omega$, it should “roll off'' at approximately 6 dB per octave (since $-20log_{10}(2) = 6.0205999 \ldots$). This is verified in the log-log plot of Fig.

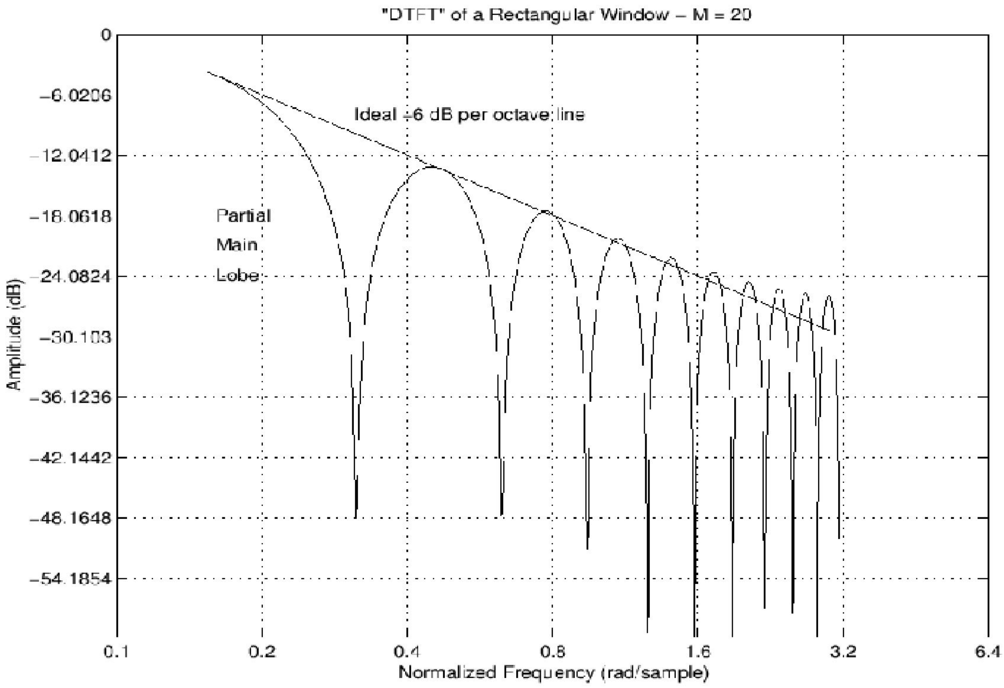


Figure : Roll-off of the rectangular-window Fourier transform.

As the sampling rate approaches infinity, the rectangular window transform (asinc) converges exactly to the sinc function. Therefore, the departure of the roll-off from that of thesinc function can be ascribed to *aliasing* in the frequency domain, due to sampling in the time domain (hence the name “asinc”).

Note that each side lobe has width $\Omega_M \triangleq 2\pi/M$, as measured between zero crossings. The main lobe, on the other hand, is width $2\Omega_M$. Thus, in principle, we should never confuse side-lobe peaks with main-lobe peaks, because a peak must be at least $2\Omega_M$ wide in order to be considered ``real''. However, in complicated real-world scenarios, side-lobes can still cause estimation errors (``bias''). Furthermore, two sinusoids at closely spaced frequencies and opposite phase can partially cancel each other's main lobes, making them appear to be narrower than $2\Omega_M$.

In summary, the DTFT of the M -sample rectangular window is proportional to the `aliased sinc function':

$$asinc_M(\omega) \triangleq \frac{\sin\left(M\frac{\omega}{2}\right)}{M.\sin\left(\frac{\omega}{2}\right)}$$

$$\approx \frac{\sin(M\pi f)}{M\pi f} \triangleq sinc(fM)$$

Thus, it has zero crossings at integer multiples of $\Omega_M \triangleq 2\pi/M$

Its main-lobe width is $2\Omega_M$ and its first side-lobe is 13 dB down from the main-lobe peak. As M gets bigger, the main-lobe narrows, giving better *frequency resolution* (as discussed in the next section). Note that the window-length M has *no effect* on side-lobe level (ignoring aliasing). The side-lobe height is instead a result of the abruptness of the window's transition from 1 to 0 in the time domain. This is the same thing as the so-called *Gibbs phenomenon*seen in truncated Fourier series expansions of periodic waveforms. The abruptness of the window discontinuity in the time domain is also what determines the side-lobe roll-off rate (approximately 6 dB per octave). The relation of roll-off rate to the smoothness of the window at its endpoints is discussed in.

Q14. Derive the condition of an FIR filter to give linear phase response. Also find the frequency response of a given FIR filter, if the number of samples, N, in its impulse response h(n) is odd. **UPTU 2012-13**

Ans. For FIR filter to have linear phase the filter must have both constant group delay and constant phase delay. To achieve both constant group delay and constant phase delay, we have

$\theta(\omega) = -\alpha\omega \; for \; -\pi \le \omega \le \pi$

Now for satisfying both conditions we have

$$h(n) = h(N-1-n),$$

i.e., The impulse response must be symmetrical about $n = \frac{N-1}{2}$

If only constant group delay is desired then we shall have

$$\theta(\omega) = \beta - \alpha\omega$$

Again, for satisfying the above condition, we have

$$h(n) = -h(N-1-n),$$

i.e., The impulse response is anti-symmetrical about $n = \frac{N-1}{2}$

Frequency response of linear phase fir filters

We know that the DTFT of a finite sequence impulse response h(n) is expressed as:

$$H(e^{j\omega}) = DTFT[h(n)]$$

$$or \qquad H(e^{j\omega}) = \sum_{n=0}^{M-1} h(n)\, e^{-j\omega nT} = |H(e^{j\omega})| e^{j\emptyset(\omega)}$$

Now if the filter length M is odd, then

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{M-3}{2}} h(n)\, e^{-j\omega nT} + h\left(\frac{M-1}{2}\right) e^{-j\omega\left(\frac{M-1}{2}\right)T} + \sum_{n=\frac{M+1}{2}}^{M-1} h(n)\, e^{-j\omega nT}$$

We know that $h(n) = h(N-1-n)$, for 0 < n < M-1

Now, applying this condition, we get

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{M-3}{2}} h(n)\left[e^{-j\omega nT} + e^{-j\omega(M-1-n)T}\right] + h\left(\frac{M-1}{2}\right) e^{-j\omega\left(\frac{M-1}{2}\right)T}$$

By substituting $k = \left(\frac{M-1}{2}\right) - n$

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{M-1}{2}\right)T} \left\{ \sum_{k=1}^{M-1} h\left(\frac{M-1}{2} - k\right)\left[e^{j\omega kT} + e^{-j\omega kT}\right] + h\left(\frac{M-1}{2}\right) \right\}$$

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{M-1}{2}\right)T} \left\{ \sum_{k=1}^{\frac{M-1}{2}} a(k) \cos \omega kT \right\}$$

Where $a(0) = h\left(\frac{M-1}{2}\right)$

$$a(k) = 2h\left(\frac{M-1}{2} - k\right) \qquad for \quad 1 \le k \le \left(\frac{M-1}{2}\right)$$

Q15. Show that the zeros of a linear phase FIR filter occurs at reciprocal location.

UPTU 2013-14

Ans. Z-transform of h(n)

$$H(z) = \sum_{k=0}^{M-1} h(k) z^{-k}$$

$$H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + \cdots + h(M-2)z^{-(M-2)} + h(M-1)z^{-(M-1)}$$

For linear phase filters, we have

$$h(n) = \pm h(M-1-n)$$

$$h(0) = \pm h(M-1)$$

$h(1) = \pm h(M-2)$ ... and so on.

$$H(z) = h(0) + h(M-1)z^{-(M-1)} + h(1)z^{-1}$$
$$+h(M-2)z^{-(M-2)} + h(2)z^{-(2)} + \cdots$$

Or

$$H(z) =$$

$$z^{-\left(\frac{M-1}{2}\right)} \left\{ h\left(\frac{M-1}{2}\right) + \sum_{n=0}^{\frac{M-3}{2}} h(n)\left[z^{\left(\frac{M-1-2n}{2}\right)} \pm z^{\left(\frac{-M-1-2n}{2}\right)}\right] \right\} \quad (for\ odd\ M)$$

$$H(z) = z^{-\left(\frac{M-1}{2}\right)} \left\{ \sum_{n=0}^{\frac{M}{2}-1} h(n)\left[z^{\left(\frac{M-1-2n}{2}\right)} \pm z^{\left(-\frac{M-1-2n}{2}\right)}\right] \right\} \quad (for\ even\ M)$$

$$H(z^{-1}) = z^{\left(\frac{M-1}{2}\right)} \left\{ \sum_{n=0}^{\frac{M}{2}-1} h(n)\left[z^{\left(-\frac{M-1-2n}{2}\right)} \pm z^{\left(\frac{M-1-2n}{2}\right)}\right] \right\}$$

Multiplying both sides by $z^{-(M-1)}$,

$$z^{-(M-1)}.H(z^{-1}) = z^{-\left(\frac{M-1}{2}\right)}\left\{\sum_{n=0}^{\frac{M}{2}-1} h(n)\left[z^{\left(-\frac{M-1-2n}{2}\right)} \pm z^{\left(\frac{M-1-2n}{2}\right)}\right]\right\} = H(z)$$

Hence

$$z^{-(M-1)}.H(z^{-1}) = H(z)$$

Which shows that roots of $H(z)$ and $H(z^{-1})$ are identical. Therefore, roots of $H(z)$ must occur in reciprocal pairs. This means that if z_1 is zero of $H(z)$, then $1/z_1$ is also a zero.

Q16. What is a Kaiser Window? In what way it is superior to another window function? Explain the procedure for designing a FIR filter using Kaiser Window.

UPTU 2014-15

Ans. The Kaiser window, also known as the Kaiser–Bessel window, was developed by James Kaiser. It is a one-parameter family of window functions used for digital signal processing.

Kaiser window allows separate control of width of the main lobe and attenuation of side lobes but with the other windows this controlling is not possible therefore Kaiser Window is superior than others.

The Kaiser window is defined by the formula:

$$w(n) = \begin{cases} \dfrac{I_0\left\{\beta\left[1-\left(\frac{n-\alpha}{\alpha}\right)^2\right]^{\frac{1}{2}}\right\}}{I_0(\beta)} & for \quad 0 \le n \le M \\ 0 & elsewhere \end{cases}$$

Where $\alpha = \frac{M}{2}$, and

- M is the length of the sequence,
- $I_0()$ is the zeroth-order modified Bessel function of the first kind,
- β is an arbitrary, non-negative real number that determines the shape of the window. In the frequency domain, it determines the trade-off between main-lobe width and side lobe level, which is a central decision in window design.

The modified Bessel function of the first kind is given as

$$I_0(x) = 1 + \frac{0.25\,x^2}{(1!)^2} + \frac{(0.25\,x^2)^2}{(2!)^2} + \frac{(0.25\,x^2)^2}{(3!)^2} + \cdots \ldots \ldots$$

For the FIR filter design using Kaiser window, minimum ripple of δ_1 and δ_2 is considered. Let the minimum ripple be represented by δ and,

$$\delta = Minimum(\delta_1, \delta_2)$$

Let the transition width $\Delta\omega$ be given as $\Delta\omega = \omega_s - \omega_p$

Attenuation $A = -20\ log_{10}\delta$

The value of β may be obtained by following empirical relations:

$$\beta = \begin{cases} 0.1102(A - 8.7) & for \quad A > 50 \\ 0.5842(A - 21)^{0.4} + 0.07886(A - 21) & for \quad 21 \le A \le 50 \\ 0 & for \quad A < 21 \end{cases}$$

And M can be obtained by following expression

$$M = \frac{A - 8}{2.285\Delta\omega}$$

The length of the filter is M+1.

Additional Questions

28. Write the expression for hamming window. **AKTU 2017-18**

Ans. Hamming Window

$$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right), & 0 \le n \le N-1 \\ 0, & otherwise \end{cases}$$

29. Design a linear phase FIR (high pass) filter of order seven with cutoff frequency π/4 radian/ sec using Hanning window. **AKTU 2017-18**

Ans. For high pass filter with cut off frequency π/4 radian/ sec

$$h_d(n) = -\frac{\sin\left(\frac{\pi}{4}(n-3)\right)}{\pi(n-3)}$$

And Hanning window function

$$w(n) = \begin{cases} \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{N-1}\right)\right], & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

for order 7

n=	$h_d(n)$=	w(n)=	h(n)= $h_d(n)$x w(n)
0	-1/3√2π	0	0
1	-1/2π	0.25	-0.6366
2	-1/√2π	0.75	-0.3001
3	-0.25	1	-0.25
4	-1/√2π	0.75	-0.3001
5	-1/2π	0.25	-0.6366
6	-1/3√2π	0	0

Thus

$$H(z) = -0.6366z^{-2} - 0.3001z^{-3} - 0.25z^{-4} - 0.3001z^{-5} - 0.6366z^{-6}$$

30. Which is more sensitive network to finite word length?

(a) Direct form-ll

(b) Cascade form

Justify your answer.

Ans. The direct form II realization requires only the layer of M or N storage elements. When compared to direct form I realisation the direct form II uses minimum number of storage elements and hence said to be a Canonic striictur JJrweves wJ' the Jc is performed sequentially, the direct form II needs two adders instead of one adder required for the direct form I.

Though the direct form I and II are commonly employed, they have two drawbacks viz (i) they lock hardware flexibility and (ii) due to finite precision arithmetic, the sensitivity of the co-efficients to quantisation effects increases with the order of the filter. This sensitivity may change the co-efficient values and hence the frequency response, thereby causing the filter to become unstable. To overcome these effects, the cascade and parallel realizations can be implemented.

31. What are the finite word length effects or error in filters?

Ans. The following are some of the finite word length effects in digital filters:

1. Errors due to quantization of input data by A/D converter.
2. Errors due to quantization of filter co-efficient.
3. Errors due to rounding the products in multiplication.
4. Errors due to overflow in addition.

5. Limit cycles

32. What is rounding effect'

Ans. Rounding is the process of reducing size of a binary number to finite size of 'b' bits such that the rounded b-bit number is closest to the original unquantized number.

The rounding process consists of truncation and addition. In rounding of a number to b-bits, first the unquantized number is truncated to b-bits by retaining the most significant b-bits Then zero or one is added to the least significant bit of the truncated number depending on the bit that is next to the least significant bit that is retained.
For Example:
0.101010 rounded to four bits is either
0.1010 or 0.1011 (Here adding one is called rounding up).
Error due to rounding: The quantization error is fixed point number due to rounding is defined as

$$\text{Rounding error, } e_r = N_r - N$$

where N_r – quantized *i.e.* rounded number

N – unquantized number.

The range of error due to rounding for all the three formats (i.e. one's complement, two's complement and sign-magnitude) of fixed point presentation is same.
In fixed point representation the range of error made by rounding a number to 'b' bits is

$$\frac{-2^{-b}}{2} \le e_r \le \frac{2^{-b}}{2}$$

Relative error due to rounding of a floating point number is given by:

$$\text{Relative error, } e_r = \frac{N_{rf} - N_f}{N_f}$$

Where N_f = Unquantized floating point binary number.
N_{rf} = Rounded floating point binary number.

33. What is fixed point representation?

Ans. In fixed point representation the bits allowed for integer part and fractional part and so the position of binary point is fixed. The main drawback of this representation is that, due to the fixed integer and fraction part, too large and to small values cannot be represented. The bit to the right represent the fractional part of the number and those to the left represent the integer part.
For Example:

The binary no. 010.11100 has the value 2.875 is decimal.

The negative numbers are represented in three different form for fixed point arithmetic

1. Sign-magnitude form.
2. One's-complement form
3. Two's-complement form.

1. Sign Magnitude form: In this form, the MSB is used to represent the given no. is positive or negative. Let 'N' be the length of binary bits, then (N-i) bit will represent magnitude and MS represent sign.

For example:

$$(2.75)_{10} = (010.1100)_2$$
→sign bit

and $$(-2.75)_{10} = (110.1100)_2$$
→sign bit

2. One's complement form: In this form the positive number is represented as in the sign magnitude notation. But the negative number is obtained by complementing all thebits of the positive number.

For example:

$(2.75)_{10} = (010.1100)_2$ and its one's complement is $(101.0011)_2 = (-2.75)_{10}$

3. Two's complement form: In this form positive numbers are represented as in sign magnitude and one's complement. The negative number is obtained by complementing all the bits of the +ve number and adding one to the least significant bit.

For example:

Consider a positive number $(2.75)_{10} = (010.\ 1100)_2$
1's complement = 101.001
Add 1 to LSB + 1
2's complement 101.0100 $(-2.75)_{10}$

34. What is floating point representation?

Ans. In floating point representation, a positive number is represented as

$$N_f = M \times 2^E$$

Where M is called mantissa and it will be in binary fraction format. The value of M will be in the range of $0.5 \le M \le 1$ and E is called exponent and it is either a positive or negative integer.

In this form, both mantissa and exponent uses one bit for representing sign. Usually the LSB is mantissa and exponent is used to represent the sign. A 'I' in the LSB represent negative sign and a '0' in the LSB represent positive sign.

The floating point representation is explained by considering a five bit mantissa and three bit exponent with a total size of eight bits. In mantissa the LSB is used to represent the sign and other four bits are used to represent a binary fraction number. In exponent the LSB is used to represent the sign and the other two bits are used to represent a binary integer number.

35. What will happen if length of windows is increased in design of FIR filters?

Ans. If length of window is increased in design of FIR filter more coefficientsneed to be calculated. More memory space used for it. More lengths of window means more accuracy in transation process.

36. Define Ripple ratio

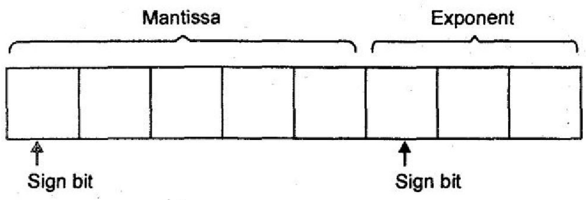
Ans. The Ripple ratio is defined as , the ratio of maximum sidelobes amplitude to the mainlobe amplitude. i.e. %RR=(maximum side lobe amplitude/main lobe amplitude)x100

37. What are the methods used to reduce Gibb's phenomenon?

Ans. There are two methods to reduce Gibb's phenomenon

1. The discontinuity between pass band and stop band in the frequency response is avoided by introducing the transition between the pass band and stop band.

2. Another technique used for the reduction of Gibb's phenomenon is by using window function that contains a taper which decays towards zero gradually instead abruptly.



For example :

$$+7_{10} = +111_2 = 0.111 \times 2^{+3}$$

$$= \underbrace{0.1110}_{+\text{ sign}} \times \underbrace{2^{+11_2}}_{+\text{ sign}}$$

$$= \underbrace{01110}_{\text{Mantissa}} \quad \underbrace{011}_{\text{Exponent}}$$

$$-7_{10} = -111_2 = 1.1110 \times 2^{+3}$$

$$= 1.1110 \times 2^{+11_2}$$

$$= \underbrace{11110}_{\text{Mantissa}} \quad \underbrace{011}_{\text{Exponent}}$$

38. What is meant by (zero input) limit cycle oscillation?

Ans. For an IIR filter implemented with infinite precision arithmetic the output should approach zero in the steady state if the input is zero and it should approach a constant value if the input is a constant. However, with an implementation using a finite length register an output can occur even with zero input. The output may be a fixed value or it may oscillate between finite positive and negative values. This effect is referred to as (zero input) limit cycle oscillation.

39. What are the assumptions made concerning the statistical independence of various noise sources that occur in realizing the filter?

Ans. Assumptions

a. for any n , the error sequence e(n) is uniformly distributed over the range
b. (-q/2) and (q/2). This implies that the mean value of e(n) is zero and its variance is
c. The error sequence e(n) is a stationary white noise source.
d. The error sequence e(n) is uncorrelated with the signal sequence x(n).

40. What is the difference between fixed point arithmetic and floating point arithmetic?

Ans.

Fixed point arithmetic	**Floating point arithmetic**
1. Fast operation	Slow operation
2. Small dynamic range	Increased dynamic range

3 Relatively economical	More expensive due to costlier hardware
4. Round-off errors occur only in addition	Round-off errors can occur with both multiplication and addition
5. Overflow occurs in addition	Overflow does not arise
6. Used in small computers	Used in larger general purpose Computers

41. What are the 3 quantization errors due to finite word length register in digital filters?

Ans. 1. Input quantization error 2. Coefficient quantization error 3. Product quantization error.

42. Explain briefly the need for scaling in the digital filter implementation.

Ans. To prevent overflow , the signal level at certain points in the digital filter must be scaled so that no overflow occurs in the adder.

43. What is limit cycles due to overflow? Or What is overflow oscillations?

Ans. The addition of two fixed point arithmetic numbers cause overflow when the sum exceeds the word size available to store the sum. This overflow caused by adder make the filter output to oscillate between maximum amplitude limits. Such limit cycles have been referred to as overflow oscillations.

44. Define 'dead band' of the filter.

Ans. The limit cycles occur as a result of quantization effect in multiplication. The amplitudes of the output during a limit cycle are confined to a range of values called the dead band of the filter.

45. Express the fraction (7/8) and (-7/8) in sign magnitude, 2's complement and 1's complement.

Ans. fraction (7/8) = (0.111) in sign magnitude , 1's complement and 2's complement

Fraction (-7/8) = (1.111) in sign magnitude

= (1.000) in 1's complement

= (1.001) in 2's complement

46. The filter coefficient H = -0.673 is represented by sign magnitude fixed point arithmetic. If the word length is 6 bits , compute the quantization error due to truncation.

Ans. (0.673) = (0.1010110…)

(-0.673) = (1.1010110…)

after truncating to 6 bits we get

(1.101011) = -0.671875

Quantization error = $x_q - x$

= (-0.671875)-(-0.673)

= 0.001125

47. Give the expression for the signal to quantization noise ratio and calculate the improvement with an increase of 2 bits to the existing bit.

Ans. SNR = 6b – 1.24dB,With an increase of 2 bits, increase in SNR is approximately 12dB.

48. Why rounding is preferred over truncation in realizing digital filters?

Ans. 1. The quantization error due to rounding is independent of the type of arithmetic.

2. The mean of rounding error is Zero. 3. The variance of rounding error signal is low.

49. What is product quantization error? Or What is round-off noise error?

Ans. Product quantization error arises at the output of a multiplier. Multiplication of a 'b' bit data with a 'b' bit coefficient results in a product having 2b bits. Since a 'b' bit register is used , the multiplier output must be rounded or truncated to 'b' bits which produces an error. This error is known as product quantization error.

50. Compare truncation with rounding errors.

Ans.

S.No.	**Truncation**	**Rounding**
1	It is the process of discarding all bits less significant than LSB that is retained	Rounding a number to b bits is accomplished by choosing the rounded result as the b bit number closest to the original number unrounded
2	For floating point system, the truncation error is seen only in mantissa	For floating point system, the truncation error is seen both in mantissa and exponent

51. What is meant by truncation?

Ans. Truncation is the process of reducing the size of binary number by discarding all bits less significant than the least significant bit that is retained. In the truncation of a binary number to b bits, all the less significant bits beyond bth bit are discarded.

52. Why is rounding preferred over truncation in realizing a digital filter?

Ans. Rounding is preferred over truncation in realizing a digital filter because of the following desirable properties:

i. The error signal due to rounding is independent of the type of binary representation

ii. Its mean is zero

iii. Rounding yields lower variance than truncation or any other method

53. A designer wants to design a discrete time low pass filter for a voice signal. The specifications are:

Passband F_p= 4 kHz, with 0.8 dB ripple

Stopband F_s= 4.5 kHz, with 50 dB Attenuation

Sampling Frequency F_{samp}= 22 kHz

Determine

1. The discrete time passband and stopband frequencies
2. The maximum and Minimum values of |H(w)|in the passband and the stopband, where H(w) is the filter frequency response.

Ans.

a) Recall the mapping from analog to digital frequency $\omega = 2\pi F / F_s$, with F_s the sampling frequency. Then the passband and stopband frequencies become $\omega_p = 2\pi 4 / 22$ rad $= 0.36\pi$ rad, $\omega_s = 2\pi 4.5 / 22$ rad $= 0.41\pi$ rad;

b) A 0.8 dB ripple means that the frequency response in the passband is within the interval $1 \pm \delta$ where δ is such that $20\log_{10}(1+\delta) = 0.8$ This yields $\delta = 10^{0.04} - 1 = 0.096$. Therefore the frequency response within the passband is within the interval $0.9035 \le |H(\omega)| < 1.096$. Similarly in the stopband the maximum value is $|H(\omega)| < 10^{-50/20} = 0.0031$

UNIT-4

(DFT & FFT)

Section A: 2 marks questions.

Q1. What is zero padding? What are its uses? **UPTU 2010-11, 2015-16**

Ans. Let the sequence x (n) has a length L. If we want to find the N-point DFT(N>L) of the sequence x(n), we have to add (N-L) zeros to the sequence x(n). This is known as zero padding. The uses of zero padding are

1) We can get better display of the frequency spectrum.

2) With zero padding the DFT can be used in linear filtering.

Q2. Determine the 4-point DFT of the sequence: **UPTU 2010-11**

X(n)=[1, 1, 0, 1]

Ans. The 4-point DFT in the matrix form

$$X_4 = [W_4].x(n)$$

$$X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+0+1 \\ 1-j+0+j \\ 1-1+0-1 \\ 1+j+0-j \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

$$Thus\ we\ get\ X_4 = \{4, 1, -1, 1\}$$

Q3. List the properties of DFT. **UPTU 2010-11**

Ans. 1) Periodicity

2) Linearity and symmetry

3) Multiplication of two DFTs

4) Circular convolution

5) Time reversal

6) Circular time shift and frequency shift

7) Complex conjugate

8) Circular correlation

Q4. Compute the N-point DFT of the signal x(n)= a^n for $0 \le n \le N-1$.

UPTU 2012-13

Ans. let x(n)= a^n

then

$$DFT\big(x(n)\big) = X(k) = \sum_{n=0}^{N-1} a^n \; e^{-\frac{j2\pi nk}{N}}, \qquad k = 0,1, \dots \dots .., N-1$$

$$X(k) = \frac{1 - a^N}{1 - ae^{-\frac{j2\pi nk}{N}}}, \qquad k = 0,1, \dots \dots \dots N-1$$

Q5. Compute X(0) if X(k) is 4-point DFT of the following sequence:

UPTU 2015-16

$$x(n) = \{1, 0, -1, 0\}$$

Ans. The 4-point DFT in the matrix form

$$X_4 = [W_4].x(n)$$

$$X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} . \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 0 - 1 + 0 \\ 1 + 0 + 1 + 0 \\ 1 + 0 - 1 + 0 \\ 1 + 0 + 1 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 2 \\ 0 \\ 2 \end{bmatrix}$$

$$Thus\ we\ get\ X_4 = \{0, 2, 0, 2\}$$

$$Therefore\ X(0) = 0.$$

Q6. If the 10-point DFT of $x(n) = \delta(n) - \delta(n-1)$ and $h(n) = u(n) - u(n-10)$ are X(k) and H(k) respectively, find the sequence w(n) that corresponds to the 10-point inverse DFT of the product H(k).X(k). **UPTU 2015-16**

Ans. Compute circular convolution of $x(n)$ and $h(n)$.

Q7. i) Compute 4-pint DFT of the following sequence using linear transformation matrix

$x(n) = \{1, 1, -2, -2\}$ **UPTU 2015-16**

Ans. The 4-point DFT in the matrix form

$$X_4 = [W_4].x(n)$$

$$X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} . \begin{bmatrix} 1 \\ 1 \\ -2 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1-2-2 \\ 1-j+2-2j \\ 1-1-2+2 \\ 1+j+2+2j \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 3-3j \\ 0 \\ 3+3j \end{bmatrix}$$

$$Thus\ we\ get\ X_4 = \{-2, 3-3j, 0, 3+3j\}$$

ii) Find IDFT x(n) from X(k) calculated in previous part. **UPTU 2015-16**

Ans. The 4-point IDFT in the matrix form

$$x(n) = \frac{1}{4}[W_4{}^*].X_4$$

$$[W_4{}^*] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$x(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} . \begin{bmatrix} -2 \\ 3-3j \\ 0 \\ 3+3j \end{bmatrix}$$

$$x(n) = \frac{1}{4}\begin{bmatrix} -2+3-3j+0+3+3j \\ -2+3j+3+0-3j+3 \\ -2-3+3j+0-3-3j \\ -2-3j-3+0+3j-3 \end{bmatrix}$$

$$x(n) = \frac{1}{4}\begin{bmatrix} 4 \\ 4 \\ -8 \\ -8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \\ -2 \end{bmatrix}$$

Q8. Find circular convolution of sequences $x_1(n)$ and $x_2(n)$ of length N=4 given by:

$x_1(n)$= [1, 2, 2, 1]

$x_2(n)$= [2, 1, 1, 2] **UPTU 2011-12**

Ans. Circular convolution

$$x_3(n) = \sum_{m=0}^{N-1} x_1(m)x_2((n-m))_N$$

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 \\ 2 & 2 & 1 & 1 \\ 1 & 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+1+2+4 \\ 4+1+1+4 \\ 4+2+1+2 \\ 2+2+2+2 \end{bmatrix} = \begin{bmatrix} 9 \\ 10 \\ 9 \\ 8 \end{bmatrix}$$

thus $x_3(n)$ = [9 10 9 8]

Q9. Find the circular convolution of the sequences: **UPTU 2010-11**

X(n)=[1, 2, 2, 1] and Y(n)=[1, 2, 3, 1] using matrix method.

Ans. Same as previous.

Q10. Obtain the circular convolution of the following: **UPTU 2010-11**

X(n)=[1, 2, 1], H(n)=[1, -2, 2]

Ans. Same as previous.

Q11. Perform the convolution of the following two sequences h(n) and x(n)

$$h(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \le n \le 2 \\ 0, & otherwise \end{cases} \quad and$$

$$x(n) = \delta(n) + \delta(n-1) + 4\delta(n-2)$$ **UPTU 2012-13**

Ans. Circular convolution

$$y(n) = \sum_{m=0}^{N-1} x(m)h((n-m))_N$$

$$x(n) = \{0, 1, 2\}$$

$$h(n) = \{1, 0.5, 0.25\}$$

$$\begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 0+1+0.25 \\ 1+0+0.5 \\ 2+0.5+0 \end{bmatrix} = \begin{bmatrix} 1.25 \\ 1.5 \\ 2.5 \end{bmatrix}$$

Therefore $y(n) = \{1.25, 1.5, 2.5\}$

Q12. Consider the sequences given by: **UPTU 2011-12, 2014-15**

$$x_1(n) = \begin{cases} 1 & 0 \le n \le 2 \\ 0 & else \end{cases}$$

$$x_2(n) = \begin{cases} 1 & 0 \le n \le 2 \\ 0 & else \end{cases}$$

Compute the linear convolution of $x_1(n)$ and $x_2(n)$ using DFT.

Ans. Length of both the sequences N_1=N_2=3

length of linear convolution N=5

thus to compute linear convolution using DFT both the sequences must have length equal to 5.

therefore x_1(n) = [1, 1, 1, 0, 0] and x_2(n) = [1, 1, 1, 0, 0]

X_1'(k) = [3, 0.5-j1.54, 0.5+j0.36, 0.5-j0.36, 0.5+j1.54] = X_2'(k)

X(k) = [9, -2.1216-j1.54, 0.1204+j0.36, 0.1204-j0.36, -2.1216+j1.54]

therefore IDFT(X(k)) = x(n) = [1, 2, 3, 2, 1]

Q13. Find the four point DFT of $x(n) = \cos\left(\frac{n\pi}{2}\right), \quad 0 \le n \le 3.$

Ans.

Given $x(n) = \cos\left(\frac{n\pi}{2}\right), \quad 0 \le n \le 3$ =[1, 0, -1, 0]

Thus DFT($x(n)$)=[0, 2, 0, 2]

Q14. Show that discrete Fourier transform can be obtained by sampling Z transform on unit circle. **UPTU 2015-16**

Ans. Discrete time signal x(n), where n is an integer.

Z-Transform

$$X(z) \triangleq \sum_{n=-\infty}^{+\infty} x(n)\, z^{-n}$$

DTFT-

$$X(e^{j\omega}) = X(z)\big|_{z=e^{j\omega}}$$

$$= \sum_{n=-\infty}^{+\infty} x(n)\, e^{-j\omega n}$$

That is how DTFT is related to z-transform

DFT:

$$X[k] = X(e^{j\omega})\big|_{\omega=\frac{2\pi k}{N}} \quad where\ x[n] = 0\ for\ n < 0\ r\ n \ge N$$

$$= \sum_{n=-\infty}^{+\infty} x[n]\,(u[n] - u[n-N])e^{-\frac{j2\pi kn}{N}}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi kn}{N}}$$

here k is an integer and u[n] is the discrete unit step function.

Q15. For the sequence $x(n) = \{0,1,2,3\}$ find, **UPTU 2014-15**

i) $x((n-2))_4$ and

ii) $x((-n))_4$

Ans. i) $x\big((n-2)\big)_4 = x(n-2+4) = x(n+2) = x'(n)$

n=	$x(n+2)$	$x'(n)$
0	$x(2)$	$x(2)$
1	$x(3)$	$x(3)$
2	$x(4)$	$x(0)$
3	$x(5)$	$x(1)$

thus x'(n) = [2, 3, 0, 1]

ii) $x\big((-n)\big)_4 = x(-n+4) = x'(n)$

n=	$x(-n+4)$	$x'(n)$
0	$x(4)$	$x(0)$
1	$x(3)$	$x(3)$
2	$x(2)$	$x(2)$
3	$x(1)$	$x(1)$

thus x'(n) = [0, 3, 2, 1]

Q16. Find inverse DFT of

i) $X(k) = \begin{cases} 3, & k = 0 \\ 1, & 1 \le k \le 5 \end{cases}$

ii) $Y(k) = e^{j2k\left(\frac{2\pi}{10}\right)} X(k)$

where $X(k)$ is 10 point DFT of $x(n)$.

Ans.

i) For N=10

X(k) = [3, 1, 1, 1, 1, 1, 0, 0, 0, 0],

IDFT (X(k)) = x(n) = [0.8, 0.2 + 0.307768j, 0.3, 0.2 + 0.072654j, 0.3, 0.2, 0.3, 0.2 - 0.072654j, 0.3, 0.2 - 0.307768j]

ii) $Y(k) = e^{j2k\left(\frac{2\pi}{10}\right)} X(k)$

from shifting property of DFT

IDFT(Y(k)) = y(n) = $x((n-2))_N$

IDFT(X(k)) = $x((n-2))_N$ = [0.3, 0.2 - 0.307768j, 0.8, 0.2 + 0.307768j, 0.3, 0.2 + 0.072654j, 0.3, 0.2, 0.3, 0.2 - 0.072654j]

Q17. Find N-point DFT of following sequences

i) $u(n) - u(n - n_0), \qquad 0 < n_0 < N$

ii) $cos^2\left(\frac{2\pi n}{N}\right), \qquad n = 0, 1, \dots\dots, N - 1.$

Ans. i) $x(n) = u(n) - u(n - n_0), \qquad 0 < n_0 < N$

$$x(n) = \begin{cases} \delta(n - n_0), & 0 < n_0 < N \\ 0, & otherwise \end{cases}$$

ii) $x(n) = cos^2\left(\frac{2\pi n}{N}\right), \qquad n = 0, 1, \dots\dots, N - 1$

$$x(n) = 2cos\left(\frac{4\pi n}{N}\right) + 1$$

$$x(n) = e^{\frac{j2\pi(2)n}{N}} + e^{-\frac{j2\pi(2)n}{N}} + e^{\frac{j2\pi(0)n}{N}}$$

Q18. Define linear convolution and its physical significance. **AKTU 2016-17**

Ans. Convolution is the method through which an output signal of an LTI system can be determined for a given input signal.

Linear convolution is the basic operation to calculate the output for any linear time invariant system given its input and its impulse response.

Q19. What is the fundamental time period of the signal x(t)=sin15πt.

AKTU 2016-17

Ans. w=15π => T=2π/15π=> thus fundamental period T=2/15 sec.

Q20. Draw a transformation matrix of size 4x4 and explain the properties of twiddle factor.

AKTU 2016-17

Ans.

Transformation matrix of size 4x4

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

properties of twiddle factor

$$W_N^{k+N} = W_N^k$$
$$W_N^{k+\frac{N}{2}} = -W_N^k$$

$$W_N^N = 1$$

$$W_N^{k+N/2} = -W_N^k$$

Q21. Calculate the DFT of the sequence s(n) = {1,2,1,3}. **AKTU 2016-17**

Ans. The 4-point DFT in the matrix form

$$X_4 = [W_4].x(n)$$

$$X_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+1+3 \\ 1-2j-1+3j \\ 1-2+1-3 \\ 1+2j-1-3j \end{bmatrix}$$

$$= \begin{bmatrix} 7 \\ j \\ -3 \\ -j \end{bmatrix}$$

$$Thus\ we\ get\ X_4 = \{7, j, -3, -j\}$$

Q22. Write the advantages of FFT over DFT. Calculate the number of multiplications needed in the calculation of DFT using FFT algorithm.

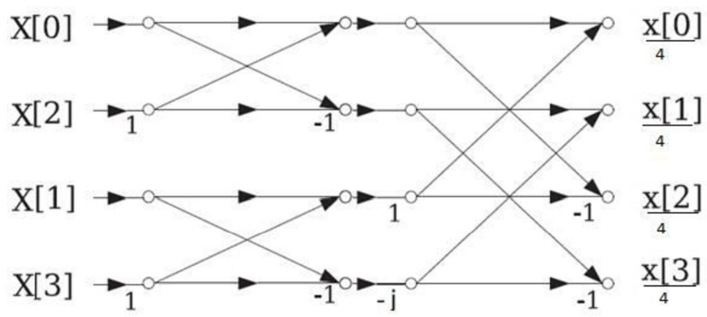
UPTU 2010-11, 2015-16

Ans. FFT is a collection of algorithms for fast computation of the DFT. Typically the number of operations required by the FFT is on the order of *N**log*N.* The most famous FFT algorithms are for the case that *N* is a power of 2, but there are FFT for prime orders and for different other factorizations.

The number of complex multiplications required using FFT is N/2log2N. If N=16, the number of complex multiplications required for direct evaluation of DFT is 256, whereas using DFT only 32 multiplications are required.

Q23. Show that the same algorithm can be used to compute IDFT of X(k) calculated in previous part. **UPTU 2015-16**

Ans. 4-point DIF IFFT butterfly diagram



X(k)= {0, 2, 0, 2} then

x(n)={1, 0, -1, 0}.

Q24. Compare the number of multiplications and additions which are needed for direct computation of DFT with those needed for radix-2 FFT algorithms.

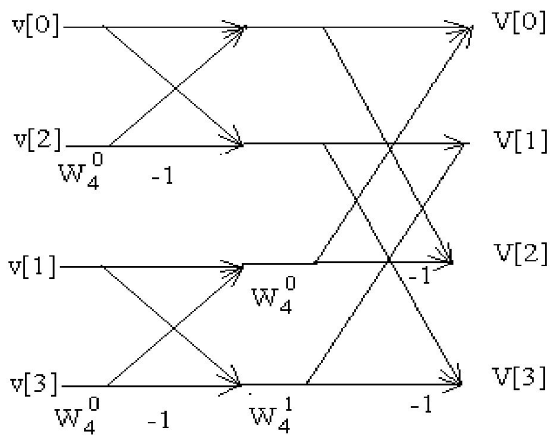
Ans.

For Radix-2 algorithm value of N is given as $N = 2^V$
Hence value of v is calculated as $V = \log_{10} N / \log_{10} 2 = \log_2 N$
Thus if value of N is 8 then the value of v=3. Thus three stages of decimation. Total number of butterflies will be Nv/2 = 12. If value of N is 16 then the value of v=4. Thus four stages of decimation. Total number of butterflies will be Nv/2 = 32.
Each butterfly operation takes two addition and one multiplication operations. Direct computation requires N^2 multiplication operation & $N^2 - N$ addition operations.

N	Direct computation		DIT FFT algorithm		Improvement in processing speed for multiplication
	Complex multiplication N^2	Complex Addition N^2-N	Complex multiplication N/2 $\log_2 N$	Complex Addition N $\log_2 N$	
8	64	52	12	24	5.3 times
16	256	240	32	64	8 times
256	65536	65280	1024	2048	64 times

Q25. Develop a DIT FFT algorithm using 4-point DFTs. **UPTU 2011-12**

Ans.



Q26. Write the expression for computation efficiency of an FFT. **AKTU 2016-17**

Ans.

For radix-2 FFT

$$X(k) = F_1(k) + W_N^k F_2(k), \qquad k = 0,1,\ldots,\frac{N}{2}-1$$

$$X(k+\frac{N}{2}) = F_1(k) - W_N^k F_2(k), \qquad k = 0,1,\ldots,\frac{N}{2}-1$$

We observe that the direct computation of $F1(k)$ requires $(N/2)2$ complex multiplications. The same applies to the computation of $F2(k)$. Furthermore, there are $N/2$ additional complex multiplications required to compute $WNkF2(k)$. Hence the computation of $X(k)$ requires $2(N/2)2 + N/2 = N\,2/2 + N/2$ complex multiplications. This first step results in a reduction of the number of multiplications from N 2 to N 2/2 + N/2, which is about a factor of 2 for N large.

The decimation of the data sequence can be repeated again and again until the resulting sequences are reduced to one-point sequences. For $N =$ 2v, this decimation can be performed v = log2N times. Thus the total number of complex multiplications is reduced to $(N/2)$log2N. The number of complex additions is Nlog2N.

Section B: 10 marks questions.

Q27. Distinguish between the following: **UPTU 2010-11**

a) Fourier Transform and Fourier Series

b) Linear and circular convolution **AKTU 2017-18**

Ans.

a) Fourier Transform and Fourier Series:

Fourier series decomposes a periodic signal into a sum of an infinite number of harmonics (sine and cosine functions) of different frequencies and amplitudes. These frequencies are discrete, not all frequencies are present. Since it is impossible to estimate an infinite series, you choose the number of terms you wish to consider, starting from the first. More the number of terms considered, closer is the series to the original signal.

Fourier transform decomposes a non-periodic signal into an infinite number of harmonics having different frequencies and amplitudes. These frequencies are continuous, no frequency is missing. The sum in this case is an integral and can be estimated for simple functions but in case of complex functions, it is estimated through numerical integration.

b) Linear convolution:

Linear convolution takes two functions of an independent variable, time, and convolves them using the convolution sum. Basically it is a correlation of one function with the time-reversed version of the other function. It is a flip, multiply, and sum while shifting one function with respect to the other. This holds in continuous time, where the convolution sum is an integral, or in discrete time using vectors, where the sum is truly a sum. It also holds for functions defined from -Inf to Inf or for functions with a finite length in time.

c) Circular convolution: Circular convolution is only defined for finite length functions (usually, maybe always, equal in length), continuous or discrete in time. In circular convolution, it is as if the finite length functions repeat in time, periodically. Because the input functions are now periodic, the convolved output is also periodic and so the convolved output is fully specified by one of its periods.

Q28. Compute the DFT Coefficients of a finite duration sequence (0,1,2,3, 0, 0, 0, 0).

UPTU 2013-14

Ans. The 8-point DFT in the matrix form

$$X_8 = [W_8].x(n)$$

$$X_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & (0.707 - j0.707) & -j & (-0.707 - j0.707) & -1 & (-0.707 + j0.707) & j & (0.707 + j0.707) \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & (-0.707 - j0.707) & j & (0.707 - j0.707) & -1 & (0.707 + j0.707) & -j & (0.707 + j0.707) \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & (-0.707 + j0.707) & -j & (0.707 + j0.707) & -1 & (0.707 - j0.707) & j & (-0.707 - j0.707) \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & (0.707 - j0.707) & j & (-0.707 + j0.707) & -1 & (-0.707 - j0.707) & -j & (0.707 - j0.707) \end{bmatrix} * \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$X_8 = \begin{bmatrix} 0 + 1 + 2 + 3 + 0 + 0 + 0 + 0 \\ 0 + (0.707 - j0.707) - 2j + 3(-0.707 - j0.707) + 0 + 0 + 0 + 0 \\ 0 - j - 2 + 3j + 0 + 0 + 0 + 0 \\ 0 + (-0.707 - j0.707) + 2j + 3(0.707 - j0.707) + 0 + 0 + 0 + 0 \\ 0 - 1 + 2 - 3 + 0 + 0 + 0 + 0 \\ 0 + (-0.707 + j0.707) - 2j + 3(0.707 + j0.707) + 0 + 0 + 0 + 0 \\ 0 + j - 2 - 3j + 0 + 0 + 0 + 0 \\ 0 + (0.707 - j0.707) + 2j + 3(-0.707 + j0.707) + 0 + 0 + 0 + 0 \end{bmatrix}$$

$$X_8 = \begin{bmatrix} 6 \\ -1.414 - j4.828 \\ -2 + j2 \\ 1.414 - j0.828 \\ -2 \\ 1.414 + j0.828 \\ -2 - j2 \\ -1.414 + j3.414 \end{bmatrix}$$

$$Thus\ we\ get\ X_8 = \begin{Bmatrix} 6, -1.414 - j4.828, -2 + j2, 1.414 - j0.828, -2, 1.414 + j0.828, \\ -2 - j2, -1.414 + j3.414 \end{Bmatrix}$$

Q29. Explain the difference between the DTFT and DFT and write the properties of DFT.

UPTU 2013-14

Ans. DTFT: Discrete Time hence time signal is in samples, Fourier Transform hence Fourier transform is a continuous function. That is why time signal is represented as x[n], n being discrete sample nos. Fourier transformed signal is still expressed in 'omega', which is not-discrete. Hence it is an infinite sequence.

DFT: Discrete time...hence time signal is in samples, the Fourier transforms are also sampled in frequency axis. Thus, its input and output sequences are both finite. DFTs are mainly used in computer based analysis as computers store data in discrete sequences with finite length. Hence storing frequency coefficients in continuous domain is not possible in digital computations.

Properties of DFT:

1. **Periodicity** :

If D.F.T(x(n))=X(k) then

D.F.T(x(n+N))=X(k)

2.**Linearity** :

If x(n) D.F.T is X(k)

Then D.F.T(a_1x_1(n)+a_2x_2(n))=a_1X_1(k)+a_1X_2(k)

3.Circular time shift

if D.F.T(x(n))=X(k) then

D.F.T(x(n-no))=w_N^{kno} X(k)

4.Circular frquency shift

if D.F.T(x(n))=X(k) then

D.F.T(w_N^{-ln} x(n))=X(k-l)

5. circular Folding

if D.F.T(x(n))=X(k) then

D.F.T (x(-n))=$X(N-K)_N$ or $X((-K))_N$

6.symmetry property

if D.F.T of x(n)=X(k)

if x(n) is real sequence then according to symmetry

we get X(k)=X*(N-K)

7.Circular convolution

Circular convolution in time domain is multiplication in frequency domain

if D.F.T x(n) =X(k)

then D.F.T(x_1(n)*x_2(n))=X_1(k)X_2(k)

8.Multiplication in time Domain

if D.F.T x(n) =X(k)

D.F.T(x_1(n)x_2(n))=(1/N)X_1(k)*X_2(k)

9.Parseval

if D.F.T x(n) =X(k)

summation of X*(n)y(n)=(1/N)summation X*(k)Y(k)

Q30. An Input Sequence x(n)= {2, 7, 0, 1, 2} is applied to a DSP system having an impulse sequence h(n) = {5, 3, 2, 1}. Determine the output sequence by (i) Linear convolution and (ii) Verify the same through circular convolution.

UPTU 2013-14

Ans. Linear Convolution (Matrix Method)

x(n) → / h(n) ↓	y(0) ⇩ 2	y(1) ⇩ 7	y(2) ⇩ 0	y(3) ⇩ 1	y(4) ⇩ 2	
5	10	35	0	5	10	⇐ y(5)
3	6	21	0	3	6	⇐ y(6)
2	4	14	0	2	4	⇐ y(7)
1	2	7	0	1	2	

Thus y(n)={10, 41, 25, 21, 20, 8, 5, 2}.

Linear Convolution (by Circular Convolution)

For circular convolution x(n)= {2, 7, 0, 1, 2, 0, 0, 0}

[zero padding]

h(n) = {5, 3, 2, 1, 0, 0, 0, 0}

[zero padding]

$$y(n) = \sum_{m=0}^{7} x(m)\, \mathrm{h}((n-m))_N$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 2 & 1 & 0 & 7 \\ 7 & 2 & 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 7 & 2 & 0 & 0 & 0 & 2 & 1 \\ 1 & 0 & 7 & 2 & 0 & 0 & 0 & 2 \\ 2 & 1 & 0 & 7 & 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 7 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 7 & 2 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 & 7 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10+0+0+0+0+0+0+0 \\ 35+6+0+0+0+0+0+0 \\ 0+21+4+0+0+0+0+0 \\ 5+0+14+2+0+0+0+0 \\ 10+3+0+7+0+0+0+0 \\ 0+6+2+0+0+0+0+0 \\ 0+0+4+1+0+0+0+0 \\ 0+0+0+2+0+0+0+0 \end{bmatrix} = \begin{bmatrix} 10 \\ 41 \\ 25 \\ 21 \\ 20 \\ 8 \\ 5 \\ 2 \end{bmatrix}$$

thus y(n)={10, 41, 25, 21, 20, 8, 5, 2}.

Q31. Compute the DFT of sequence $b^n \cos(an)$ and show that

UPTU 2011-12, 2014-15

$$IDFT[X(k-m)] = W_N^{mn} DFT[x(k)]$$

OR

Calculate the DFT of x(n) = Cos an [HINT: put b=1] **AKTU 2016-17**

Ans.

let x'(n)= b^n

then

$$DFT\big(x'(n)\big) = X'(k) = \sum_{n=0}^{N-1} b^n \; e^{-\frac{j2\pi nk}{N}}, \quad k = 0,1, \dots \dots .., N-1$$

$$X'(k) = \frac{1-b^N}{1-be^{-j2\pi nk/N}}, \quad k = 0,1, \dots \dots \dots N-1$$

and $\cos(an) = \frac{1}{2}(e^{jan} + e^{-jan})$

thus x(n) = $\frac{1}{2}(e^{jan} + e^{-jan})b^n$

for $l = aN/2\pi$

by using frequency shift property

$$X(k) = \frac{1}{2}[X'\big((k-l)\big)_N + X'\big((k+l)\big)_N$$

From frequency shift property

$$IDFT[X(k-m)] = W_N^{mn} DFT[x(k)]$$

Q32. State the circular shift property of DFT.

Ans. Circular Shift Property of DFT.

Time Shift

$$If \; \tilde{x}[n] \overset{DFT}{\longleftrightarrow} \tilde{X}[k]$$

$$then \quad \tilde{x}[n-m] \overset{DFT}{\longleftrightarrow} e^{-j2\pi km/N} \tilde{X}[k]$$

Proof:

$$DFT\{\tilde{x}[n-m]\} \triangleq \sum_{n=0}^{N-1} \tilde{x}[n-m]e^{-\frac{j2\pi kn}{N}}$$

$$Let\ [n-m] = l,$$

$$then\ n|_{n=0}^{N-1}, \qquad l|_{-m}^{N-1-m}, \qquad n = l+m$$

$$DFT\{\tilde{x}[n-m]\} = \sum_{l=-m}^{N-1-m} \tilde{x}[n]e^{-\frac{j2\pi k(l+m)}{N}}$$

$$= e^{-\frac{j2\pi km}{N}} \sum_{l=-m}^{N-1-m} \tilde{x}[l]e^{-\frac{j2\pi kl}{N}}$$

$$\equiv e^{-\frac{j2\pi km}{N}} \tilde{X}[k]$$

b. Frequency Shift

$$If\ \tilde{x}[n] \overset{DFT}{\longleftrightarrow} \tilde{X}[k]$$

$$then \quad e^{j2\pi nl/N}\ \tilde{x}[n] \overset{DFT}{\longleftrightarrow} \tilde{X}[k-l]$$

Proof:

$$DFT\{\ e^{j2\pi nl/N}\ \tilde{x}[n]\} \triangleq \sum_{n=0}^{N-1} e^{j2\pi nl/N}\ \tilde{x}[n]e^{-\frac{j2\pi kn}{N}}$$

$$= \sum_{n=0}^{N-1} \tilde{x}[n]e^{-\frac{j2\pi(k-l)n}{N}}$$

$$\equiv \tilde{X}[k-l]$$

Q33. State and prove the "circular time shift" property of DFT.

Ans. Same as previous.

Q34. Distinguish between DIT and DIF algorithm. Draw the flow graph of a two point radix-2 DIT and DIF FFT. **UPTU 2010-11**

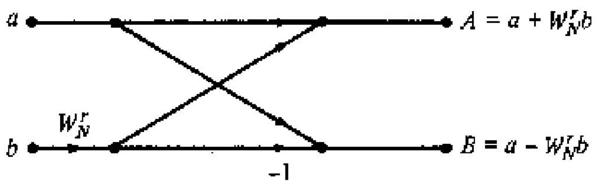
Ans. Differences:

1. For DIT, the input is bit reversal while the output is in natural order, whereas for DIF, the input is in natural order while the output is bit reversed.

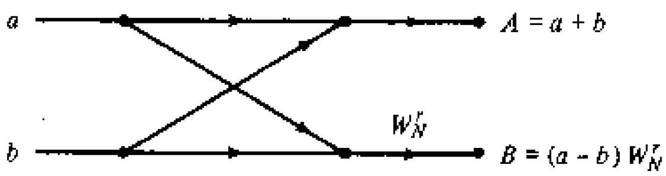
2. The DIF butterfly is slightly different from the DIT butterfly, the difference being that the complex multiplication takes place after the add-subtract operation in DIF.

Similarities:

Both algorithms require same number of operations to compute the DFT. Both algorithms can be done in place and both need to perform bit reversal at some place during the computation.



2-point radix-2 DIT FFT



2-point radix-2 DIF FFT

Q35. Compute the DFT of the sequence $x(n) = \cos(n\pi)/2$ whose N=4 using DIF FFT algorithm. **UPTU 2010-11**

Ans. For N = 4

$$x(0) = \frac{\cos(0)}{2} = \frac{1}{2}$$

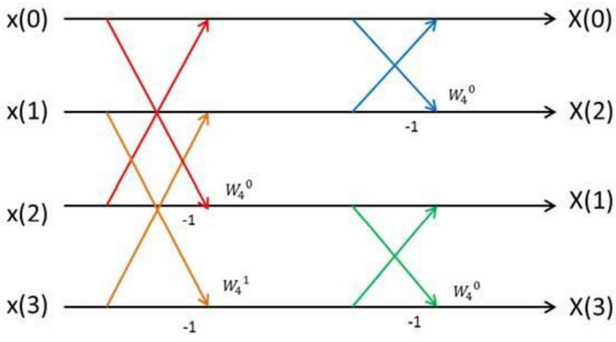
$$x(1) = \frac{\cos(\pi)}{2} = \frac{-1}{2}$$

$$x(2) = \frac{\cos(2\pi)}{2} = \frac{1}{2}$$

$$x(3) = \frac{\cos(3\pi)}{2} = \frac{-1}{2}$$

$$x(n) = \left\{\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right\}$$

The butterfly diagram for 4-point DIF FFT can be given as



On solving the above butterfly we get

$$X(0) = \left(\frac{1}{2} + \frac{1}{2}\right) + \left(-\frac{1}{2} - \frac{1}{2}\right) = 0$$

$$X(2) = \left(\frac{1}{2} + \frac{1}{2}\right) - \left(-\frac{1}{2} - \frac{1}{2}\right) = 2$$

$$X(1) = \left(\frac{1}{2} - \frac{1}{2}\right) + \left(-\frac{1}{2} + \frac{1}{2}\right)(-j) = 0$$

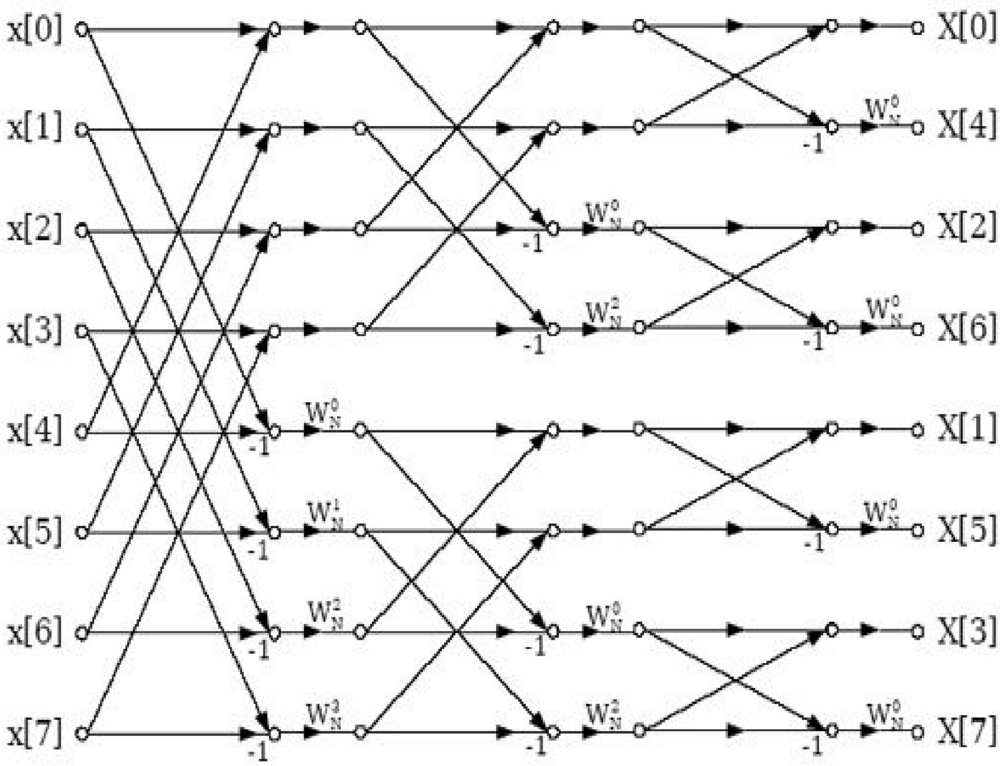
$$X(3) = \left(\frac{1}{2} - \frac{1}{2}\right) - \left(-\frac{1}{2} + \frac{1}{2}\right)(-j) = 0$$

Thus $X(k) = \{0, 0, 2, 0\}$

Q36. Given x(n) = (n + 1) and N = 8. Find X(K) using DIF FFT algorithm.

UPTU 2013-14

Ans. 8-point DIF FFT butterfly diagram



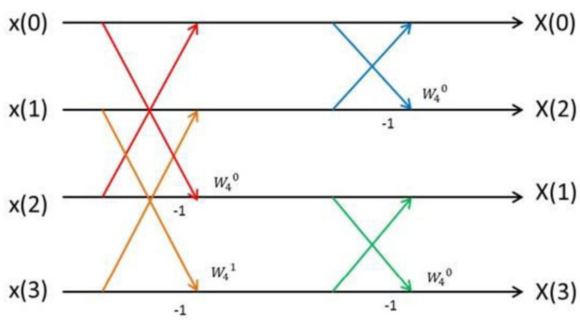
x(n)={1, 2, 3, 4, 5, 6, 7, 8}

then X(k)= {36, -4+9.66i, -4+4i, -4+1.66i, -4, -4-1.66i, -4-4i, -4-9.66i}

Q37. Compute 4-point DFT of the following sequence using DIF algorithm

$$x(n) = \cos\frac{n\pi}{2}$$ **UPTU 2015-16**

Ans. 4-point DIF FFT butterfly diagram



x(n)={1, 0, -1, 0} then

X(k)= {0, 2, 0, 2}

Q38. Compute the DFT of following 8-point sequence using 4-point radix-2 algorithm.

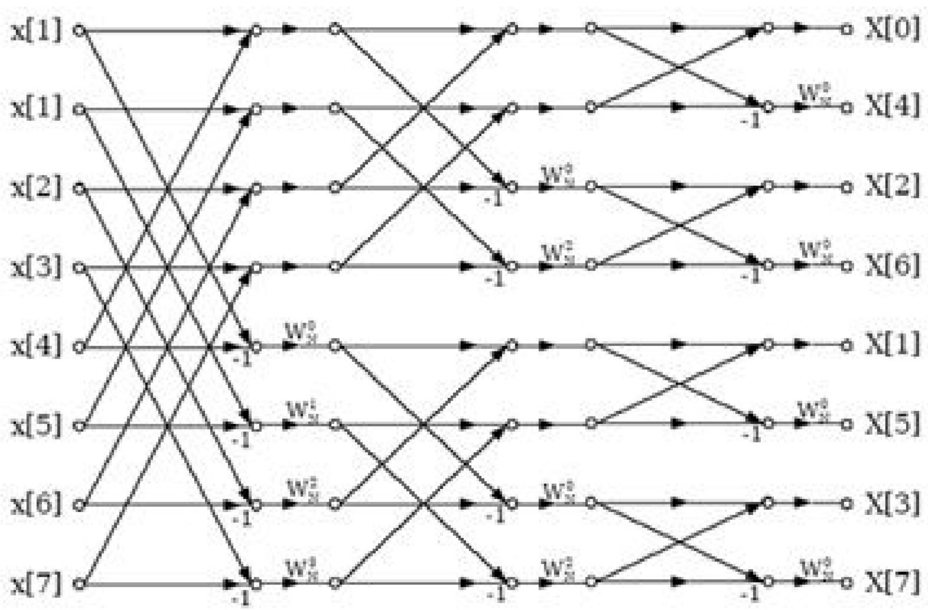
$x(n) = \{2, 2, 2, 2, 1, 1, 1, 1\}$ **UPTU 2015-16**

Ans. Do it Yourself

Q39. Draw the flow graph of an 8-point DIF algorithm and mention different expressions.

UPTU 2012-13, AKTU 2016-17, 2017-18

Ans.

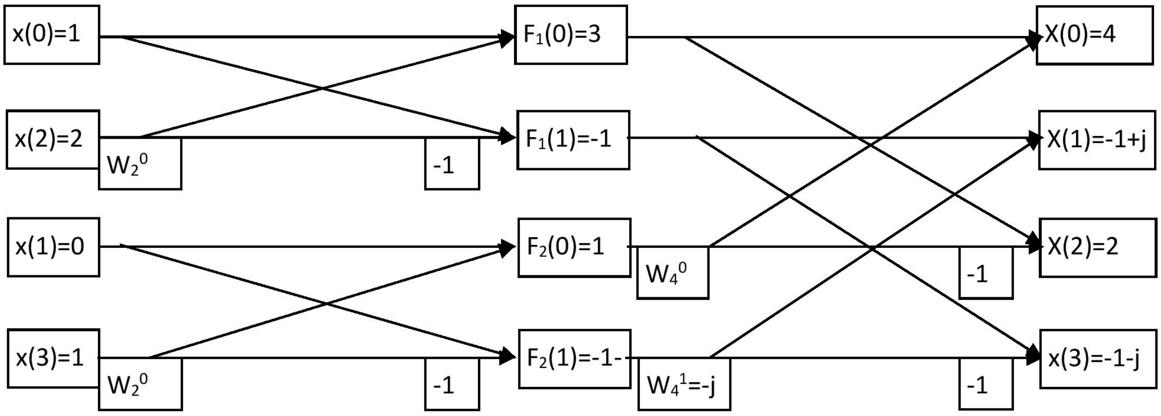


$$X(2k) = \sum_{m=0}^{\frac{N}{2}-1} \left[x(n) + (-1)^{2k} x\left(n + \frac{N}{2}\right) W_N^{2kn}\right]$$

$$X(2k+1) = \sum_{m=0}^{\frac{N}{2}-1} \left[x(n) + (-1)^{(2k+1)} x\left(n + \frac{N}{2}\right) W_N^{(2k+1)n}\right]$$

Q40. Determine the 4-point DFT of the sequence $x(n) = \{1,0,2,1\}$ using decimation in time FFT algorithm. **UPTU 2014-15**

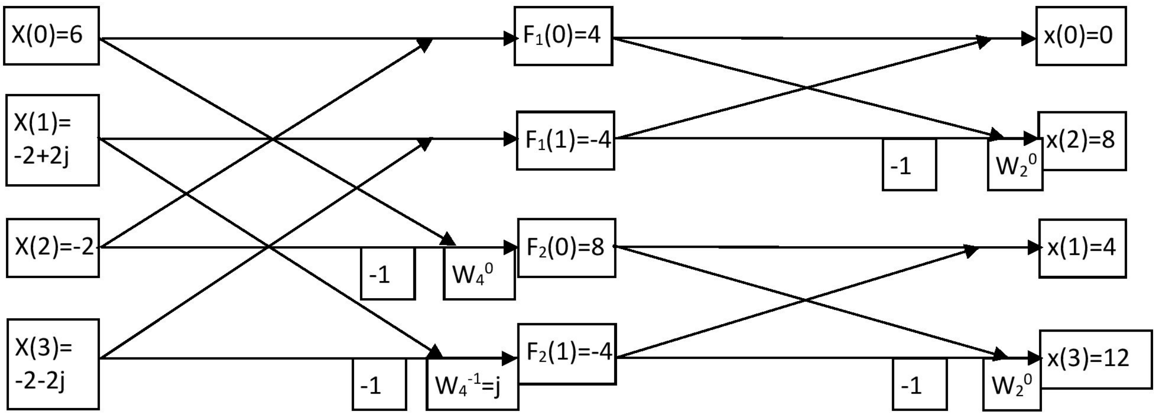
Ans.



Thus X(k) = [4, -1+j, 2, -1-j]

Q41. Find the inverse-DFT of the sequence $X(k) = \{6, -2+2j, -2, -2-2j\}$ using efficient computation algorithm. **UPTU 2014-15, AKTU 2017-18**

Ans.



Thus x(n) = (1/4)[0, 4, 8, 12] = [0, 1, 2, 3]

Q42. Develop a DIT FFT algorithm using 4 point DFTs for the case $N = 4^v$.

UPTU 2011-12, 2013-14

Ans. Radix-4 FFT Algorithm

When the number of data points N in the DFT is a power of 4 (i.e., $N = 4^v$), we can, of course, always use a radix-2 algorithm for the computation. However, for this case, it is more efficient computationally to employ a radix-r FFT algorithm.

Let us begin by describing a radix-4 decimation-in-time FFT algorithm briefly. We split or decimate the N-point input sequence into four subsequences, $x(4n)$, $x(4n+1)$, $x(4n+2)$, $x(4n+3)$, $n = 0, 1, \ldots, N/4-1$.

$$X(p,q) = \sum_{l=0}^{3}\left[W_N^{lq} F(l,q)\right] W_4^{lp}$$

$$F(l,q) = \sum_{m=0}^{(N/4)-1} x(l,m) W_{N/4}^{mq}$$

$$p = 0,1,2,3; \quad l = 0,1,2,3; \quad q = 0,1,2,\ldots,\frac{N}{4}-1$$

and

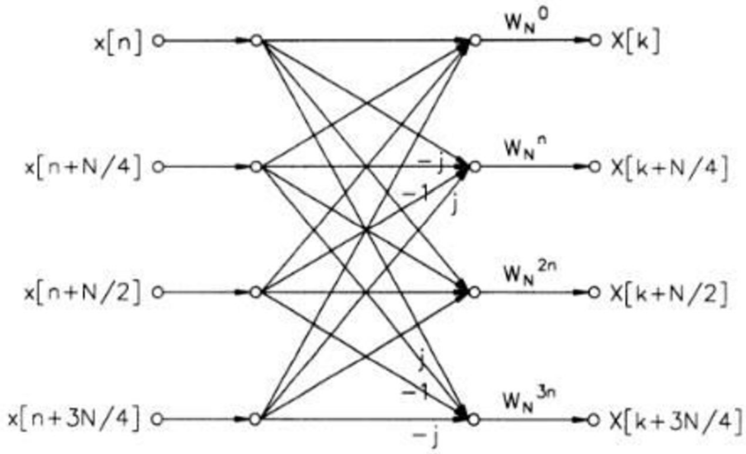
$$x(l,m) = x(4m+1)$$

$$X(p,q) = X(\frac{N}{4}p + q)$$

Thus the four $N/4$-point DFTs $F(l, q)$obtained from the above equation are combined to yield the N-point DFT. The expression for combining the $N/4$-point DFTs defines a radix-4 decimation-in-time butterfly, which can be expressed in matrix form as

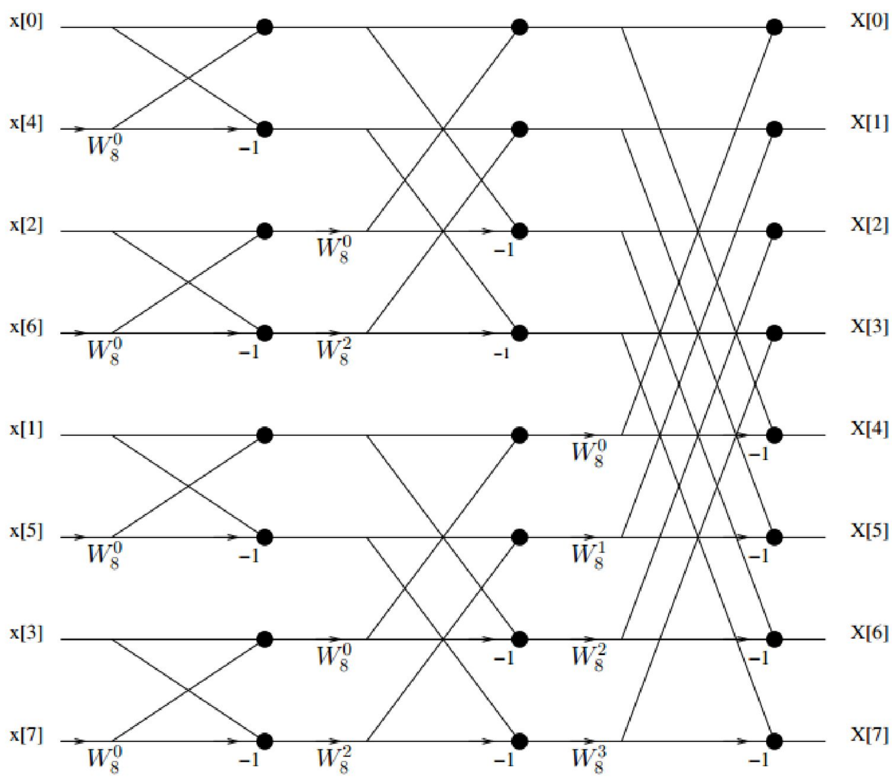
$$\begin{bmatrix} X(0,q) \\ X(1,q) \\ X(2,q) \\ X(3,q) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} W_N^0 F(0,q) \\ W_N^q F(1,q) \\ W_N^{2q} F(2,q) \\ W_N^{3q} F(3,q) \end{bmatrix}$$

The radix-4 butterfly is depicted in Figure. Note that each butterfly involves three complex multiplications, since $W_N^0 = 1$, and 12 complex additions.



Q43. Given that x(n) = {2, 2, 2,2, l, l, 1, l}. Compute 8-point FFT using decimation-in-time method. **UPTU 2012-13**

Ans. 8-point DIT FFT butterfly diagram



x(n) = {2, 2, 2,2, l, l, 1, l}

X(k)= {12, 1-2.4i, 0, 1-0.4i, 0, 1+0.4i, 0, 1+2.4i}.

Section C: 15 marks questions.

Q44. Find 10- point DFT of the following sequences: **UPTU 2015-16**

i) $x(n) = \delta(n) + \delta(n-5)$

ii) $x(n) = u(n) + u(n-6)$

Ans.

$$DFT\big(x(n)\big) = X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}$$

i) $x(n) = \{1, 0, 0, 0, 0, 1, 0, 0, 0, 0\}$

For $N = 10$, $k = 0, 1, 2, \ldots..9$

And $n = 0, 1, 2, \ldots..9$

Thus

$$X(0) = x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6) + x(7) + x(8) + x(9)$$
$$= 1 + 0 + 0 + 0 + 0 + 1 + 0 + 0 + 0 + 0 = 2$$

$$X(1) = x(0) + x(1)e^{-\frac{j2\pi}{10}} + x(2)e^{-j2\pi*\frac{2}{10}} + x(3)e^{-j2\pi*\frac{3}{10}} + x(4)e^{-j2\pi*\frac{4}{10}} + \cdots$$
$$x(5)e^{-j2\pi*5/10} + x(6)e^{-j2\pi*6/10} + x(7)e^{-j2\pi*7/10} + x(8)e^{-j2\pi*8/10}$$
$$+ x(9)e^{-j2\pi*9/10} = 1 + 0 + 0 + 0 + 0 - 1 + 0 + 0 + 0 + 0 = 0$$

Similarly remaining values can be calculated.

ii) $x(n) = \{1, 1, 1, 1, 1, 1, 2, 2, 2, 2\}$

Calculate DFT same as above.

Q45. Prove that multiplication of the DFT's of two sequences is equivalent to the circular convolution of the two sequences in the time domain. **UPTU 2015-16**

Ans. Circular Convolution

$x(n)$ and $h(n)$ are two finite sequences of length N with DFTs denoted by $X(k)$ and $H(k)$, respectively. Let us form the product

$$W(k) = X(k)H(k),$$

and determine the sequence $w(n)$ of length N for which the DFT is $W(k)$.

First, extend $x(n)$ and $h(n)$ to periodic sequences with period N, $\tilde{x}(n)$ and $\tilde{h}(n)$respectively. Then, the periodic convolution of $\tilde{x}(n)$and $\tilde{h}(n)$ corresponds to multiplication of the corresponding periodic sequences of Fourier series coefficients

$$\tilde{w}(n) = \sum_{m=0}^{N-1} \tilde{x}(m)\,\tilde{h}(n-m) \leftrightarrow \widetilde{W}(k) = \tilde{X}(k)\widetilde{H}(k)$$

The periodic sequence $\tilde{w}(n)$ for which the DFS coefficients are $\widetilde{W}(k)$ corresponds to the periodic extension of the finite length sequence $w(n)$ with period N. We can recover $w(n)$ by extracting one period of $\tilde{w}(n)$:

$$w(n) = \tilde{w}(n)\mathbb{R}_N(n)$$

$$= \sum_{m=0}^{N-1} \tilde{x}(m)\,\tilde{h}(n-m)$$

$$= \sum_{m=0}^{N-1} x(n)\,h((n-m))_N$$

This operation is called circular convolution and denoted

$$w(n) = x(n) \oplus h(n).$$

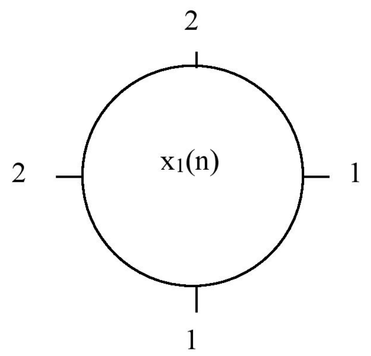
Q46. State and prove the "circular convolution" property of DFT. **AKTU 2017-18**

Ans. Same as Previous.

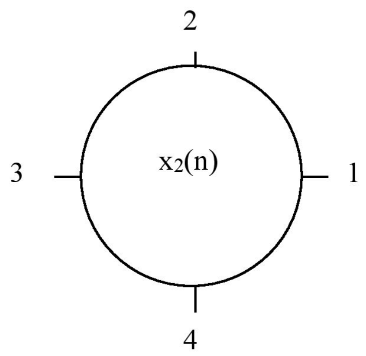
Q47. Find circular convolution of the following sequences using concentric circle method

$x_1(n) = \{1, 2, 2, 1\}$ $x_2(n) = \{1, 2, 3, 4\}$ **UPTU 2015-16**

Ans.



$x_1(n) = \{1, 2, 2, 1\}$



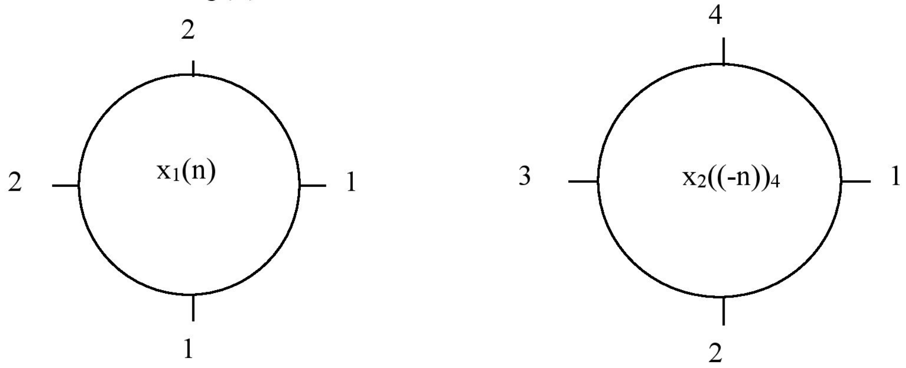
$x_2(n) = \{1, 2, 3, 4\}$

Circular Convolution

$$x_3(m) = \sum_{n=0}^{3} x_1(n)\, x_2((m-n))_4$$

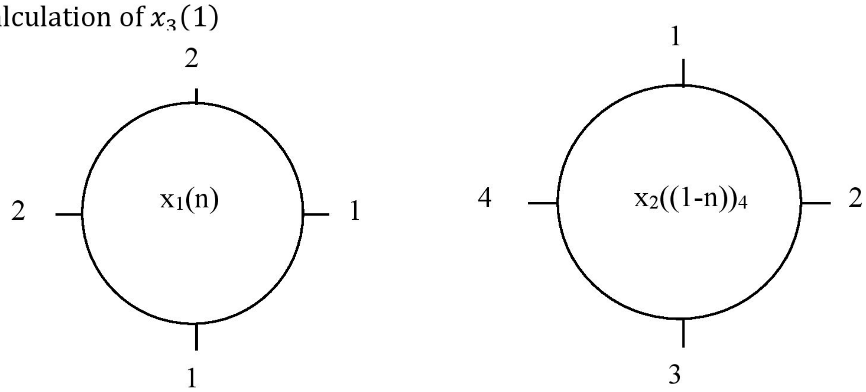
Concentric Circle Method:

Calculation of $x_3(0)$



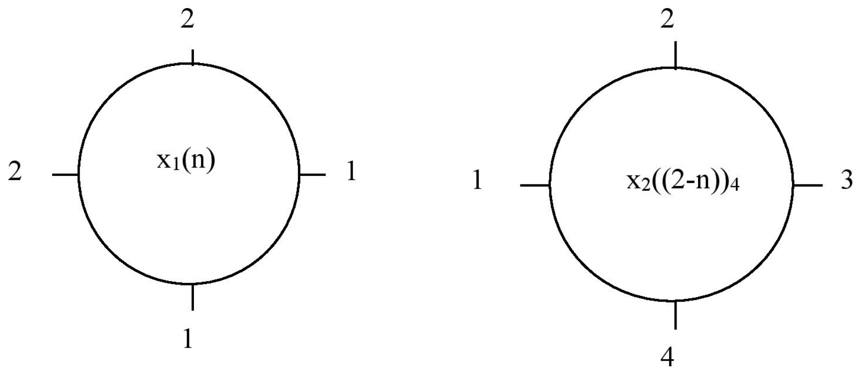
$$x_3(0) = 1*1 + 2*4 + 2*3 + 1*2 = 1 + 8 + 6 + 2 = 17$$

Calculation of $x_3(1)$



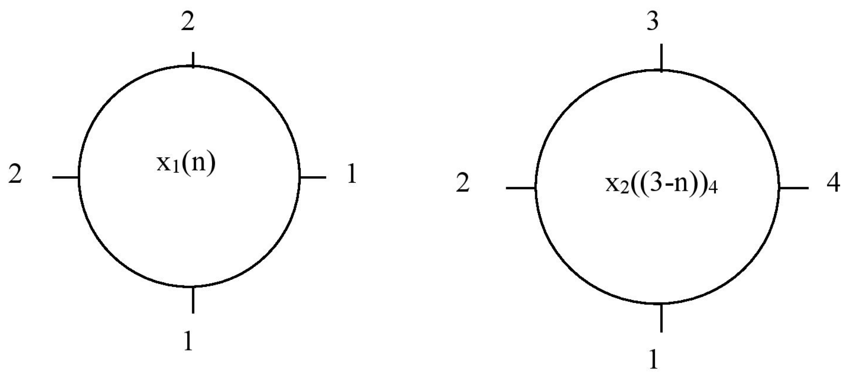
$$x_3(1) = 1*2 + 2*1 + 2*4 + 1*3 = 2 + 2 + 8 + 3 = 15$$

Calculation of $x_3(2)$



$x_3(0) = 1 * 3 + 2 * 2 + 2 * 1 + 1 * 4 = 3 + 4 + 2 + 4 = 13$

Calculation of $x_3(3)$



$$x_3(0) = 1 * 4 + 2 * 3 + 2 * 2 + 1 * 1 = 4 + 6 + 4 + 1 = 15$$

Therefore $x_3(n) = \{17, 15, 13, 15\}$.

Q48. Prove that DFT of a finite length sequence is the same as samples of DTFT in one period.

Ans.

To relate the DFT and DTFT, we will need to truncate the DTFT to a finite range of *N* samples. In doing so, one would like to insure that the power of the signal in the excluded (truncated) portion is insignificant, e.g. one would not want to approximate the DSFT with the DFT for $x(n) = \sin(4\pi n)$, which will have significant power in any truncated portion. This is equivalent to saying that the DTFT of *x* cannot have any delta functions.

Next, one needs to determine if the range of $x(n)$ should be [0…*N*-1] or [-*N*/2…*N*/2-1]. That will determine if ifftshift needs to be used. Now suppose we have a function that is zero for all negative *n* and we determine a length *N* over which we'd like to examine. We can now approximate $X(\omega)$ as $X_d(k)$, where $\omega = 2\pi k/N$.

Note that the continuous $X(\omega)$ is now discretized or sampled. Locations of samples are either $\omega \in (2\pi/N)$ $[0...N-1]$ or $\omega \in (2\pi/N)$ $[N/2...N/2-1]$ depending of whether fftshift has been used.

This range is approximately [0..2π] or [-π…π], respectively. This introduces a second criterion on selection *N*. If *X* (ω) appears to be sampled too coarsely, then N should be increased, even if it means adding zeros to ends of the definition of *x* (know as zero padding).

The inverse is relationship is also tricky. First, let's assume that we have defined a continuous *X* (ω). Obviously, *X* (ω) cannot have any delta functions as these cannot be represented on the computer. This can be sampled at locations ω ∈ (2π/ *N)* [0...*N* −1] or ω ∈(2π/ *N)* [*N* / 2...*N* / 2 −1] andone them implements the inverse DFT (ifft) to get a signal *x*(*n*). The tricky part arises from the observation that the DFT is DFS are closely related. This means that by sampling *X* (ω) , we have created a periodic *x*(*n*). This creates an ambiguity as to whether *x*(*n*) is defined for *n* in [0…*N*-1] or [-*N*/2…*N*/2-1]. If we know (e.g. based on causality or whatever) that the output will be zero for negative *n*, then we can use the output of the DFT for *n* in [0…*N*-1]. If we don't know, it might be wise to use fftshift and define *n* for [-*N*/2…*N*/2-1]. *N* should be chosen so that *x*(*n*) had decayed to near zero at -*N*/2 and *N*/2-1.

Q49. State and prove the linearity and symmetry properties of DFT.

Ans. Linearity: Let $\{x_0, x_1, \ldots x_{N-1}\}$ and $\{y_0, y_1, \ldots y_{N-1}\}$ be two sets of discrete samples with corresponding DFT's given by X(m) and Y(m). Then DFT of sample set $\{x_0+y_0, x_1+y_1, \ldots x_{N-1}+y_{N-1}\}$ is given by X(m)+Y(m)

Proof:

$$X(m) = \sum_{n=0}^{N-1} x_n e^{\frac{-j2\pi nm}{N}} \quad ; \quad Y(m) = \sum_{n=0}^{N-1} y_n e^{\frac{-j2\pi nm}{N}}$$

$$X(m) + Y(m) = \sum_{n=0}^{N-1} (x_n + y_n) e^{\frac{-j2\pi nm}{N}}$$

Symmetry:

If the samples x_n are real, then extracting in frequency domain X(0)…….X(N-1) seems counter intuitive; because, from N bits of information in one domain (time), we are deriving 2N bits of information in frequency domain. This suggests that there is some redundancy in computation of X(0)…….X(N-1). As per DFT symmetry property, following relationship holds.

X(N-m)=X*(m) m=0, 1,……., N-1, where symbol * indicates complex conjugate.

Proof:

$$X(m) = \sum_{n=0}^{N-1} x_n e^{\frac{-j2\pi nm}{N}}$$

$$X(N-m) = \sum_{n=0}^{N-1} x_n e^{\frac{-j2\pi n(N-m)}{N}}$$

$$= \sum_{n=0}^{N-1} x_n e^{\frac{j2\pi nm}{N}} e^{-j2\pi n}$$

Since $e^{-j2\pi n} = 1$

$$X(N-m) = \sum_{n=0}^{N-1} x_n e^{\frac{j2\pi nm}{N}}$$

$$= \sum_{n=0}^{N-1} (x_n e^{\frac{-j2\pi nm}{N}})^*$$

$$= [\sum_{n=0}^{N-1} x_n e^{\frac{-j2\pi nm}{N}}]^*$$

$$= (X(m))^* = X^*(m)$$

Q50. Compute the Circular convolution of two discrete time sequences x1(n) = {1, 2, 1, 2} and x2(n) = { 3, 2, 1, 4 } **AKTU 2016-17**

Ans. Circular convolution

$$x_3(n) = \sum_{m=0}^{N-1} x_1(m) x_2((n-m))_N$$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 3+4+1+8 \\ 6+2+2+4 \\ 3+4+1+8 \\ 6+2+2+4 \end{bmatrix} = \begin{bmatrix} 16 \\ 14 \\ 16 \\ 14 \end{bmatrix}$$

thus $x_3(n)$ = [16 14 16 14]

Q51. Determine the 4-point discrete time sequence from its DFT X(k) = { 4, 1-j, -2, 1+j } **AKTU 2016-17**

Ans. The 4-point IDFT in the matrix form

$$x(n) = \frac{1}{4} [W_4{}^*]. X_4$$

$$[W_4{}^*] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$x(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix}$$

$$x(n) = \frac{1}{4}\begin{bmatrix} 4+1-j-2+1+j \\ 4+j+1+2-j-1 \\ 4-1+j-2-1-j \\ 4-j+1+2+j-1 \end{bmatrix}$$

$$x(n) = \frac{1}{4}\begin{bmatrix} 4 \\ 6 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 3/2 \\ 0 \\ 3/2 \end{bmatrix}$$

Q52. Derive the relation between DFT and Z-transform of a discrete time sequence s(n).

AKTU 2016-17

Ans.

Relation between DFT and Z-transform:

Let us consider that $X(Z)$ is the Z-transform of the time domain sequence $x(n)$ is defined as

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

its ROC includes unit circle. Consider that $X(Z)$ is sampled at the N evenly spaced point on the circle with radius one. Therefore,

$$z_k = e^{j2\pi k/N}; \qquad k = 0, 1, 2, 3, \ldots N-1.$$

Hence,

$$X(k) = X(Z)\Big|_{z = e^{\frac{j2\pi k}{N}}} = \sum_{n=-\infty}^{\infty} x(n)\, e^{-\frac{j2\pi k n}{N}}$$

It is identical to the Fourier Transform of Sequence $x(n)$ *i.e.* $X(\omega)$ obtained at the N evenly spaced frequencies

$$\omega_k = \frac{2\pi k}{N}, \qquad k = 0, 1, 2, \ldots N-1$$

If the signal sequence $x(n)$ is of finite duration length 'N', then the Z-transform is given by

$$X(Z) = \sum_{n=0}^{N-1} x(n)\, z^{-n}$$

Put the IDFT relation to $x(n)$, we obtain,

$$X(Z) = \sum_{n=0}^{N-1} \left\{ \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi nk}{N}} \right\} z^{-n}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) \sum_{n=0}^{N-1} \left(e^{\frac{j2\pi k}{N}} z^{-1} \right)^{n}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) \frac{\left(e^{\frac{j2\pi k}{N}} z^{-1} \right)^{0} - \left(e^{\frac{j2\pi k}{N}} z^{-1} \right)^{N-1+1}}{1 - e^{\frac{j2\pi k}{N}} z^{-1}}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) \left\{ \frac{1 - \left(e^{j2\pi k/N} z^{-1} \right)^{N}}{1 - e^{j2\pi k/N} z^{-1}} \right\}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) \left\{ \frac{1 - z^{-N}}{1 - e^{j2\pi k/N} z^{-1}} \right\}$$

or

$$X(Z) = \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{X(k)}{1 - e^{j2\pi k/N} z^{-1}}$$

Therefore, by putting $z = e^{j\omega}$, we obtain,

$$X(\omega) = \frac{1 - e^{-j\omega N}}{N} \sum_{k=0}^{N-1} \frac{X(k)}{1 - e^{-j(\omega - 2\pi k/N)}}$$

Q53. Develop a radix-4 DIT FFT algorithm for evaluating the DFT for N = 16 and hence determine the 16 point DFT of the sequence x(n)={0, l, 0, 1, 0, 1,0, 1,0, 1,0, 1,0, l, 0, l}

UPTU 2013-14

Ans. When the number of data points N in the DFT is a power of 4 (i.e., $N = 4^v$), we can, of course, always use a radix-2 algorithm for the computation. However, for this case, it is more efficient computationally to employ a radix-r FFT algorithm.

Let us begin by describing a radix-4 decimation-in-time FFT algorithm briefly. We split or decimate the N-point input sequence into four subsequences, $x(4n)$, $x(4n+1)$, $x(4n+2)$, $x(4n+3)$, $n = 0, 1, \ldots, N/4-1$.

$$X(p,q) = \sum_{l=0}^{3} \left[W_N^{lq} F(l,q)\right] W_4^{lp}$$

$$F(l,q) = \sum_{m=0}^{(N/4)-1} x(l,m) W_{N/4}^{mq}$$

$$p = 0,1,2,3; \quad l = 0,1,2,3; \quad q = 0,1,2,\ldots,\frac{N}{4}-1$$

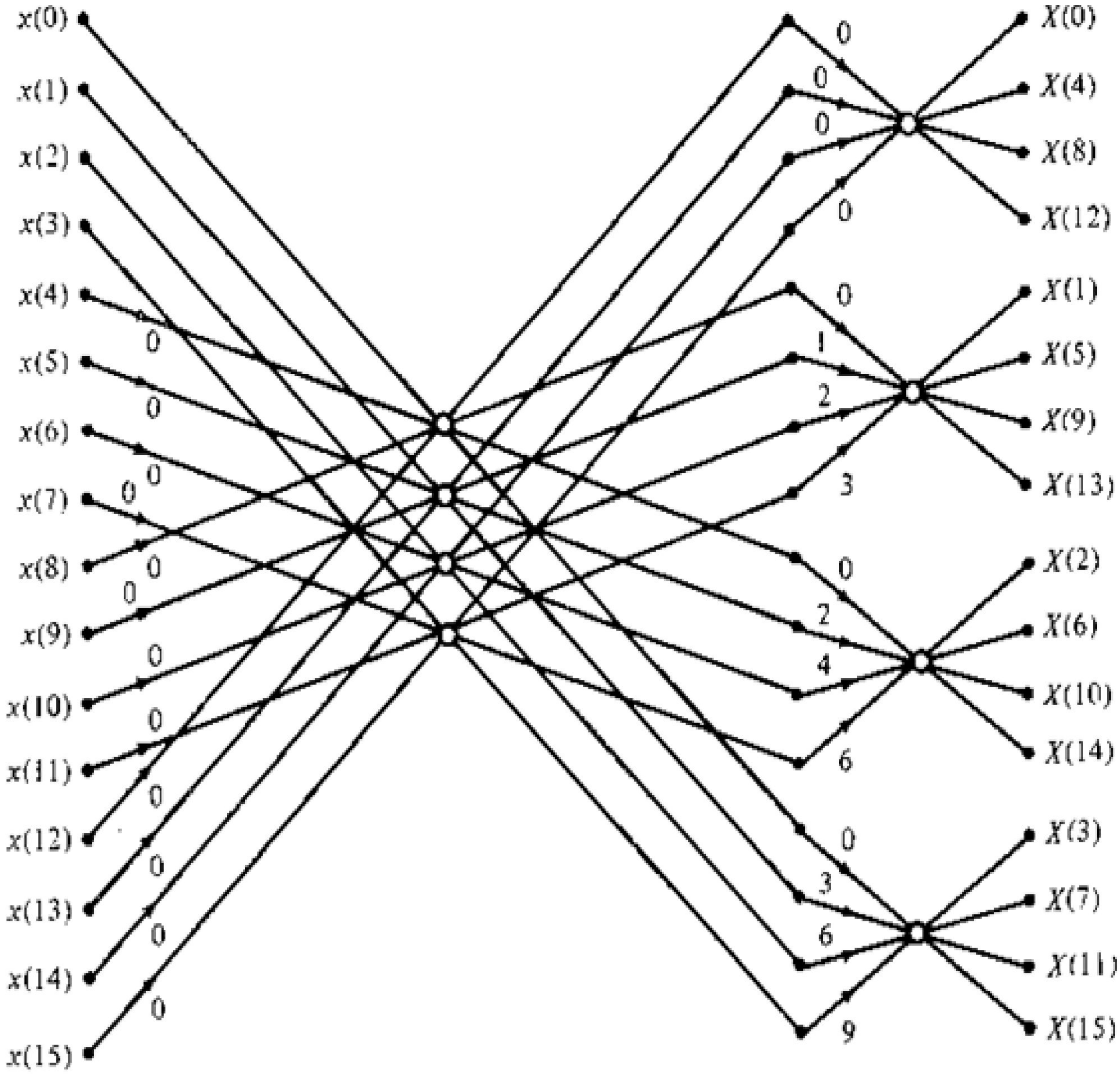
and

$$x(l,m) = x(4m+1)$$

$$X(p,q) = X(\frac{N}{4}p + q)$$

By performing the additions in two steps, it is possible to reduce the number of additions per butterfly from 12 to 8. This can be accomplished ty expressing the matrix of the linear transformation mentioned previously as a product of two matrices as follows:

$$\begin{bmatrix} X(0,q) \\ X(1,q) \\ X(2,q) \\ X(3,q) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -j \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & j \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} W_N^0 F(0,q) \\ W_N^1 F(1,q) \\ W_N^2 F(2,q) \\ W_N^3 F(3,q) \end{bmatrix}$$



x(n)={0, l, 0, 1, 0, 1,0, 1,0, 1,0, 1,0, l, 0, l} then

X(k)= {8, 0, 0, 0, 0, 0, 0, 0, -8, 0, 0, 0, 0, 0, 0, 0}

Q54. Use Radix-2 DIT algorithm for efficient computation of 8-point DFT of $x(n) = 2^n$.

UPTU 2015-16, AKTU 2016-17

Ans. x(n)= {1, 2, 4, 8, 16, 32, 64, 128} rest same as Section-B-Q10.

Q55. What do you mean by FFT? Differentiate between DIT and DIF FFT algorithm. State and prove the symmetry and periodicity properties of complex exponential sequence W_M^k. Explain how these properties are used in FFT algorithm.

UPTU 2011-12, 2014-15

Ans. The Fast Fourier Transform is an algorithm optimization of the DFT—Discrete Fourier Transform. The "discrete" part just means that it's an adaptation of the Fourier Transform, a continuous process for the analog world, to make it suitable for the sampled digital world. The results of the FFT are the same as with the DFT; the only difference is that the algorithm is optimized to remove redundant calculations. In general, the FFT can make these optimizations when the number of samples to be transformed is an exact power of two, for which it can eliminate many unnecessary operations.

S. No.	DIT FFT	DIF FFT
1	DIFFFT algorithms are based upon decomposition of the input sequence into smaller and smaller sub sequences.	DIFFFT algorithms are based upon decomposition of the output sequence into smaller and smaller sub sequences.
2	In this input sequence x(n) is splitted into even and odd numbered samples	In this output sequence X(k) is considered to be splitted into even and odd numbered samples
3	Splitted operation is done on time domain sequence.	Splitting operation is done on frequency domain sequence.
4	In DIT FFT input sequence is in bit reversed order while the output sequence is in natural order.	In DIFFFT, input sequence is in natural order. And DFT should be read in bit reversed order.

There are three properties of twiddle factor W_N

1) $W_N^{k+N} = W_N^k$ (Periodicity Property)
2) $W_N^{k+N/2} = -W_N^K$ (Symmetry Property)
3) $W_N^2 = W_{N/2}$.

Q56. Show that the output data is in bit reversed order for the decimation-in-frequency algorithm for N=8. **UPTU 2011-12, 2012-13, 2014-15**

Ans. BIT REVERSAL

For 8 point DIT DFT input data sequence is written as x(0), x(4), x(2), x(6), x(1), x(5), x(3), x(7) and the DFT sequence X(k) is in proper order as X(0), X(1), X(2), X(3), X(4), x(5), X(6), x(7). In DIF FFT it is exactly opposite. This can be obtained by bit reversal method.

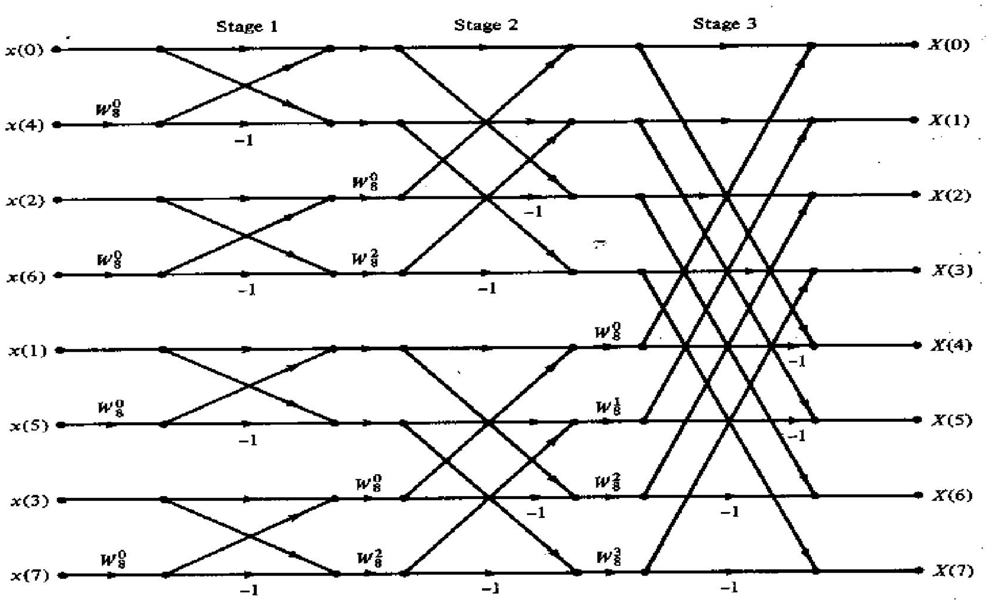
Decimal	Memory Address x(n) in binary (natural order)			Memory Address in bit reversed order			New Address in decimal
0	0	0	0	0	0	0	0
1	0	0	1	1	0	0	4
2	0	1	0	0	1	0	2
3	0	1	1	1	1	0	6
4	1	0	0	0	0	1	1
5	1	0	1	1	0	1	5
6	1	1	0	0	1	1	3
7	1	1	1	1	1	1	7

Table shows first column of memory address in decimal and second column as binary. Third column indicates bit reverse values. As FFT is to be implemented on digital computer simple integer division by 2 method is used for implementing bit reversal algorithms. Flow chart for Bit reversal algorithm is as follows.

Q57. Draw the signal flow graph of an 8-point DFT computation using DIT algorithm and mention the corresponding expressions of signals at various nodes.

UPTU 2012-13

Ans.



$$X(k) = F_1(k) + W_N^k F_2(k)$$

$$X(k + N/2) = F_1(k) - W_N^k F_2(k)$$

Additional Questions

54. If x (n) = {6, 5, 4, 3} what will be x ((2-n))4. **AKTU 2017-18**

Ans. x ((2-n))4= x'(n) = {4, 5, 6, 3}

55. What is the DFT of δ (n)? **AKTU 2017-18**

Ans. DFT(δ (n)) = 1

56. Define Time Reversal of a sequence in DFT. **AKTU 2017-18**

Ans. Time reversal property of DFT states that

$$x((-n))_N \xrightarrow[DFT]{} X(-k) = X^*(k)$$

57. What is twiddle factor in DFT? **AKTU 2017-18**

Ans. Twiddle Factor $W_N = e^{-j2\pi/N}$

58. Use the 4 point DFT and IDFT to determine circular convolution of the following sequence:
x (n) = {1, 2, 3, 1} **AKTU 2017-18**
h (n) = {4, 3. 2, 2}

Ans. The 4-point DFT of x(n)

$$X(k) = [W_4].x(n)$$

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+3+1 \\ 1-2j-3+j \\ 1-2+3-1 \\ 1+2j-3-j \end{bmatrix} = \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix}$$

Thus we get $X(k) = \{7, -2-j, 1, -2+j\}$

similarly 4-point DFT of h(n) is $H(k) = \{11, 2-j, 1, 2+j\}$

multiplication of the two DFTs X(k) and H(k)

$$Y(k) = \{77, -5, 1, -5\}$$

IDFT of Y(k)

$$y(n) = [W_4]^*. Y(k)/N$$

$$y(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \cdot \begin{bmatrix} 77 \\ -5 \\ 1 \\ -5 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 77-5+1-5 \\ 77-5j-1+5j \\ 77+5+1+5 \\ 77+5j-1-5j \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 68 \\ 76 \\ 88 \\ 76 \end{bmatrix}$$

therefore the circular convolution $y(n) = \{17, 19, 22, 19\}$

59. Determine the 8-point DFT of the following sequence using DIF FFT algorithm:
x (n) = {1, 2. 3. 4} **AKTU 2017-18**

Ans. The new 8-point sequence after zero padding becomes

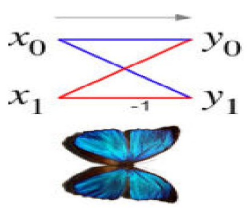
x (n) = {1, 2. 3. 4, 0, 0, 0, 0}

rest is same as Q37

60. Write a short notes on the following: **AKTU 2017-18**
(i) Butterfly Computation (ii) Inplace Computation (iii) Bit reversal

Ans.

i) In the context of fast Fourier transform algorithms, a butterfly is a portion of the computation that combines the results of smaller discrete Fourier transforms (DFTs) into a larger DFT, or vice versa (breaking a larger DFT up into subtransforms). The name "butterfly" comes from the shape of the data-flow diagram in the radix-2 case.



Signal-flow graph

In the context of fast Fourier transform algorithms, a butterfly is a portion of the computation that combines the results of smaller discrete Fourier transforms (DFTs) into a larger DFT, or vice versa (breaking a larger DFT up into subtransforms). The name "butterfly" comes from the shape of the data-flow diagram in the radix-2 case, as described below. The earliest occurrence in print of the term is thought to be in a 1969 MIT technical report. The same structure can also be found in the Viterbi algorithm, used for finding the most likely sequence of hidden states.

Most commonly, the term "butterfly" appears in the context of the Cooley–Tukey FFT algorithm, which recursively breaks down a DFT of composite size n = rm into r smaller transforms of size m where r is the "radix" of the transform. These smaller DFTs are then combined via size-r butterflies, which themselves are DFTs of size r (performed m times on corresponding outputs of the sub-transforms) pre-multiplied by roots of unity (known as twiddle factors).

ii) In-place computation:- To compute the elements p and q of the mth array , it is required to have elements in the p and q of the (m-1) array. If Xm(p) and Xm(q) are stored in the same register as Xm-1(p) and Xm-1(q) respectively ,it is possible to implement the above computation with only N array of complex storage registers. This kind of computation is commonly referred to as In-place computation.

iii) "Bit reversal" is just what it sounds like: reversing the bits in a binary word from left to right. Therefore the MSBs become LSBs and the LSBs become MSBs. But what does that have to do with FFTs? Well, the data ordering required by radix-2 FFTs turns out to be in "bit reversed" order, so bit-reversed indexes are used to combine FFT stages. It is possible (but slow) to calculate these bit-reversed indices in software; however, bit reversals are trivial when implemented in hardware. Therefore, almost all DSP processors include a hardware bit-reversal indexing capability (which is one of the things that distinguishes them from other microprocessors.)

61. Determine the circular convolution of the following sequences and compare the results with linear convolution: **AKTU 2017-18**

x (n) = (1,2,3,4)

h (n) = (1,2,1)

Ans. For circular convolution h(n)= (1,2,1,0)

Circular convolution

$$\begin{bmatrix}1 & 4 & 3 & 2\\2 & 1 & 4 & 3\\3 & 2 & 1 & 4\\4 & 3 & 2 & 1\end{bmatrix}\begin{bmatrix}1\\2\\1\\0\end{bmatrix}=\begin{bmatrix}1+8+3+0\\2+2+4+0\\3+4+1+0\\4+6+2+0\end{bmatrix}=\begin{bmatrix}12\\8\\8\\12\end{bmatrix}$$

Thus circularly convolved sequence y(n) = (12, 8, 8, 12)

Linear Convolution

x(n) → / h(n) ↓	y(0) 1	y(1) 2	y(2) 3	y(3) 4	
1	1	2	3	4	⇐ y(4)
2	2	4	6	8	⇐ y(5)
1	1	2	3	4	⇐ y(6)

Thus linearly convolved sequence y(n)={1, 4, 8, 12, 11, 4}.

Sum of all the elements of linear and circular convolutions are equal but the length of the two convolutions is different. Length of circular convolution is the same as the length of two sequences i.e. N, whereas the length of linear convolution is equal to L+M-1, where L and M are the length of the two given sequences.

62. The first five point of the 8-point DFT of a real valued sequence are: {0.25, 0.125 - j0.3018, 0, 0.125 - j0.0518, 0}. Determine the remaining three points.

AKTU 2017-18

Ans. In case of real valued sequence

$X(k) = X^*(N-k)$

Thus $X(5) = X^*(3) = 0.125 + j0.0518$

$X(6) = X^*(2) = 0$ and

$X(7) = X^*(1) = 0.125 + j0.3018$

UNIT-5

(Multirate Digital Signal Processing)

Section-A- 2 marks questions

1. **What is multi-rate signal processing?**

Ans. The process of converting a signal from a given rate to a different rate is called sampling rate conversion. Systems that employ multiple sampling rates in the processing of digital signals are called multi rate digital signal processing.

2. **Define down-sampling.**

Ans. The process of reducing the sampling rate by an integer factor(D) is called decimation of the sampling rate. It is also called down sampling by factor(D).Decimator consists of decimation filter to band limit the signal and down sampler to decrease the sampling rate by an integer factor (D).

3. **What is meant by up-sampling?**

Ans. Increasing sampling rate of a signal by an integer factor I is known as Interpolation or up-sampling. An increase in the sampling rate by an integer factor I may be done by interpolating (I-1) new samples between successive values of the signals.

4. **If the spectrum of a sequence x(n) is $X(e^{jw})$,then what is the spectrum of a signal down-sampled by a factor 2.**

Ans. $y(n) = 1/2\ [1 + e^{j\pi n/2}]$

5. **If the Z transform of a sequence x(n) is X(z), then what is the Z- transform of a sequence down sampled by a factor M?**

Ans. $Y(z) = \Sigma\ v(m)\ [1/M\ \Sigma\ e^{j2\pi mk/D}]\ z^{-m/D}$

6. **What must be done to avoid aliasing?**

Ans.

(i) Pre alias filter must be used to limit band of frequencies of the signal to f_m Hz.

(ii) Sampling frequency 'f_s' must be selected such that $f_s > 2\ f_m$

7. **What is the need for anti aliasing filter prior to down sampling?**

Ans. Anti aliasing filter is used to avoid aliasing caused by down sampling the signal x(n)

8. **What is the need for anti imaging filter after up sampling a signal?**

Ans. Anti imaging filter removes the unwanted images that are that are yielded by up sampling.

9. **Mention the areas in speech processing?**

Ans. Encoding, Synthesis and recognition.

10. **Give some applications of speech synthesis**.

Ans.

(i) Automatic Intercept system for telecommunications
(ii) Announcement systems about time, weather warnings etc.

(iii) Voice output of electronic mail, voice alarms
(iv) Reading machines for dumb or blind
(v) Data base enquiry services in railways, flight information, stock prices, quotations, sports scores etc.

11. Give some applications of multi rate signal processing.

Ans.

a. Design of phase shifters
b. Interfacing of digital systems with different sampling rates
c. Implementation of narrow band LPF & implementation of Digital Filter Bank
d. Sub band coding of speech signals & Quadrature mirror filter
e. Trans multiplexers & Over sampling of A/D and D/A conversion

12. Mention the applications of speech coding.

Ans. Digital transmission like telephony, narrow band cellular radio, military communications and secrecy missions, voice mail sent on telephone networks, voice encryption, integrated voice and data transmission over packet networks.

13. Give the main classification of speech sounds.

Ans. Voiced sounds and unvoiced sounds

14. Give some methods of analysis of speech in DSP.

Ans. Short time Fourier analysis, Homomorphic Filtering and Linear prediction

15. Give the range of sampling rate for speech coding.

Ans. Sampling rate : 8 KHz

16. What are the methods of speech coding techniques?

Ans. Waveform coding:

(i)Pulse code modulation (ii) Adaptive pulse code modulation (iii)Linear Predictive coding (iv) Frequency domain coding.

17. What is sub-band coding?

Ans. The speech signal is applied to an analysis filter bank consisting of a set of Q band pass filters. This digital filtration divides the speech signal into a non overlapping frequency bands. These filter banks are contiguous in frequency. Hence, by additive recombination of the set of sub band signals, one can approximately generate the original speech signal.

18. What is meant by Image Enhancement?

Ans. Image enhancement is to improve the appearance of images for human perception by making some features of the image like edges or contrast, more prominent relative to others. It is done for the purpose of image analysis or for display

19. What is the use of adaptive filters?

Ans. Adaptive filters are capable of adjusting their co-efficient continuously during transmission of data and this is done by operating on the received signal in accordance with some algorithm. These filters are used for adaptive equalization of channel output and adaptive prediction in adaptive Differential pulse code modulation

20. Give the advantages of digital recording.

Ans.

a. A high signal to noise ratio limited by A to D conversion accuracy
b. Absence of wow- flutter speed variation
c. Elimination of harmonic distortion at upper signal extremity
d. Removal of amplitude variations caused by changes in tape magnetization
e. Avoiding inter channel cross talk

21. What is meant by interpolation?

Ans. The process of increasing the sampling rate by an integer factor I i.e., up sampling by I,is called interpolation.

22. Write the input output relationship for a decimator.

Ans. The input output relationship for a decimator is given by,

Fy = Fx/D

where, Fy - sampling rate of the output signal

Fx - sampling rate of the input signal

D - decimation factor

23. Write the input output relationship for an interpolator.

Ans. The input output relationship for an interpolator is given by,

Fy = IFx

where, Fy - sampling rate of the output signal

Fx - sampling rate of the input signal

I - Interpolation factor

24. For the signal f(t) = 5 cos (5000*t) + sin2 (3000*t), determine the minimumsampling rate for recovery without aliasing.

Ans. From the given signal function f(t) the frequency f1 = 2500 Hz and f2 = 1500 Hz.

From sampling theorem, the sampling rate Fs % 2f

The minimum sampling rate without aliasing, Fs = 2(2500)

Fs = 5000 Hz

25. Differentiate between anti-aliasing and anti-imaging filters.

Ans.

S.No.	Anti-aliasing filter	Anti-imaging filter
1	It is a filter used before a signalsampler, to restrict the bandwidthof a signal to approximately satisfythe sampling theorem	It is used to construct a smoothanaloguesignal from a digitalinput, as in the case ofa digital to analog converter (DAC) orother sampled data outputdevice
2	Used at the input side of a digitalsignal processing system	Used at the output side of a digital signalprocessing system

26. How is sampling rate converted by a factor I/D?

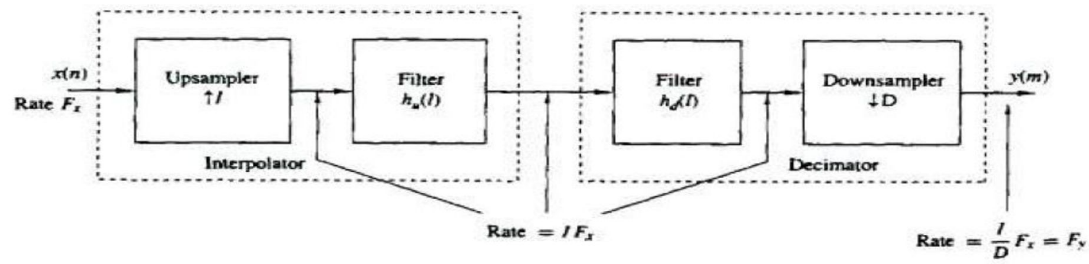
Ans. The sampling rate can be converted by a factor I/D by cascade connection ofinterpolator and decimator.

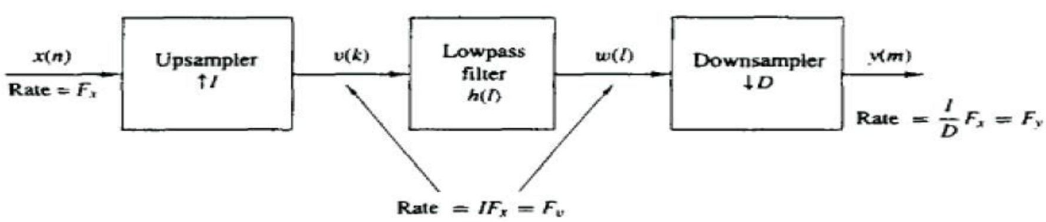
27. What are called polyphase filters?

Ans. Polyphase is a way of doing sampling rate conversion that leads to very efficientimplementations.But more than that, it leads to very general viewpoints that are useful inbuilding filter banks. "Polyphase Filters" is often incorrectly taken to mean some specialkind of filter instead, it is merely a special structure that is handy when using filters inmultirate settings.

28. Draw the schematic for sampling rate conversion by a factor I/D. Express thespectrum of output sequence.

Ans. Schematic for sampling rate conversion by a factor I/D





29. What is echo cancellation?

Ans. The term echo cancellation is used in telephony to describe the process of removingecho from a voice communication in order to improve voice quality on a telephone call.In addition toimproving subjective quality, this process increases the capacity achievedthrough silence suppression by preventing echo from traveling across a network.Echo cancellation involves first recognizing the originally transmitted signal that reappears,with some delay, in the transmitted or received signal. Once the echo isrecognized, it can be removed by 'subtracting' it from the transmitted or received signal.This technique is generally implemented using a digital signal processor (DSP).

30. What is Quadrature Mirror Filter (QMF)?

Ans. Quadrature Mirror Filter is a filter whose magnitude response is mirror image ofanother filter about $\Omega = \frac{\pi}{2}$. Together, these filters are known as the Quadrature MirrorFilter pair.

A filter $H_1(z)$will be quadrature mirror filter of$H_0(z)$ if $H_1(z) = H_0(-z)$

The filter responses are symmetric about $\Omega = \frac{\pi}{2}$.

31. What are the sections of QMF?

Ans. The sections of QMF are:

i. Analysis section

ii. Synthesis section

32. What do you understand by “sample rate conversion” and “multirate” digital signal processing systems ?

Ans. **Multi rate DSP :**

• Multi rate simply means "multiple sampling rates". A multi rate DSP system uses multiple sampling rates within the system. Whenever a signal at one rate has to be used by a system that expects a different rate, the rate has to be increased or decreased, and some processing is required to do so. Therefore "Multi rate DSP" really refers to the art or science of changing sampling rates.

• Multi-rate processing finds use in signal processing systems where various sub-systems with differing sample or clock rates need to be interfaced together. At other times multi-rate processing is used to reduce computational overhead of a system. For example, an algorithm requires k operations to be completed per cycle. By reducing the sample rate of a signal or system by a factor of M, the arithmetic bandwidth requirements are reduced from kfs operations to kfs/M operations per second.

• The most immediate reason is when you need to pass data between two systems which use incompatible sampling rates. For example, professional audio systems use 48 kHz rate, but consumer CD players use 44.1 kHz; when audio professionals transfer their recorded music to CDs, they need to do a rate conversion.

• But the most common reason is that multirate DSP can greatly increase processing efficiency (even by orders of magnitude!), which reduces DSP system cost..

Multirate consists of:

• Decimation: To decrease the sampling rate,

• Interpolation: To increase the sampling rate, or,

• Resampling: To combine decimation and interpolation in order to change the sampling rate by a fractional value that can be expressed as a ratio. For example, to resample by a factor of 1.5, you just interpolate by a factor of 3 then decimate by a factor of 2 (to change the sampling rate by a factor of 3/2=1.5.)

Section-B- 5 marks questions

33. Explain the principle of sampling rate conversion for multirate signal processing.

34. Explain in detail the poly phase structures for decimation and interpolation filters.

35. Explain the design of narrow band filters in detail.

36. With necessary equations and diagrams, discuss about the interpolation and decimation in multirate signal processing.

37. Design one-stage and two-stage interpolators to meet the following specification:

I = 20

Input sampling rate : 10000 Hz

Pass band : $0 \leq F \leq 90$

Transition ban : $90 \leq F \leq 100$

Ripple : δ1 = 10-2 and δ2 = 10-3

Section-C- 10 marks questions

38. Explain the design procedures of narrow band filters.
39. What is aliasing? Explain the aliasing effect in digital filters with suitable waveforms. Mention the methods of eliminating the aliasing effect.
40. Write short notes on Sampling & Sampling Rate alteration.
41. Explain the different interpolation techniques with suitable examples.

What is Nyquist sampling theorem? Explain Decimation by a factor of 'D'and Interpolation by a factor of 'L' with suitable examples